THE DETECTION OF INTERACTION EFFECTS

A REPORT ON A COMPUTER PROGRAM FOR THE SELECTION OF OPTIMAL COMBINATIONS OF EXPLANATORY VARIABLES


BY
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## PREFACE

The motivation for the development of the computer program described in this report comes from two sources. First, is the belief that the multivariate statistical techniques in common usage are often inadequate for the analysis of the rich body of data from a cross section sample survey, and second is the conviction that a large-scale digital computer can be used for more than just a high-speed adding machine.

Modern data-collection techniques produce a wide variety of data. These range from classifications through rankings to continuous variables which sometimes approach near-normality in their distributions. Generally, they contain a variable amount of error, with little evidence as to its size or extent of randomess. When data come from a complex probability sample, serious questions arise as to the proper application of statistical tests of significance which usually assume simple random sampling models. Intercorrelations between explanatory variables make their effects difficult to assess and, when complex interaction effects and departures from linearity are present, the analyst has a difficult task indeed. Finally, some explanatory variables are logically prior to others, in that they can affect them, but cannot, in turn be affected.

Given the large amount of data, the essence of research strategy is to put some restrictions on the process in order to make it manageable. The more theoretical or statistical assumptions one is willing to impose on the data, the more the complexity of the analysis can be reduced. But the restrictions imposed in advance through the use of most conventional multivariate techniques cannot be tested. It appears to us to be desirable not to impose advance assumptions of linearity, absence of interaction and normality, yet to be able to consider the simultaneous effects of thirty or forty variables.

We have tried to break away from the habit of asking the question, "What is the effect of $x$ on $y$ when everything else is held constant?" This has been replaced with, "What do I need to know most in order to reduce predictive error a maximum amount?"

This is the type of question that might be asked by a research scientist working in a substantive area in which theory is not yet very precise. Once he receives an answer, he may well ask, "Now that I know this, what additional information would help to reduce predictive error still further?" and so on. He would certainly ask other questions as his results came back, but he would be unable to explore very many variables in this fashion without the aid of powerful machine techniques.

We have felt that one approach to the development of more satisfactory multivariate analysis techniques might be to start with the analysis strategy a scientist might use in exploring the system of relationships among a few variables, formalize it, and extend it to more variables by simulating the formal model on the computer.

The strategy implemented in what follows is admittedly very limited, and deliberately so, but it seems to work. What is clear is that sequential data-analysis strategies far more sophisticated than the present one can be programmed, and that the modern computer can provide an extension of the analytic capabilities of the research scientist in addition to being an extension of his pencil.

We would like to express our appreciation to the various people and organizations who have made important contributions to this work:
i. to Kathleen Goode, Keith Mather and David Schupp of the Institute for Social Research Data-Processing staff, and especially to Wen Chao Hsieh who did the programming.
ii. to Professors L. J. Savage and William Ericson for their advice and help. Professor Ericson's Note on Partitioning for Maximum Between Sum of Squares, a proof of the sufficiency of the partitioning algorithm, is incorporated herein.
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## CHAPTER I

## THE PROBLEM AND THE PROGRAM

Section 1.1 Abstract and Indexing Description
This report describes a computer program written in MAD and UMAP, for the IBM 7090, operating under the University of Michigan Executive System. The program is useful in studying the interrelationships among a set of up to 37 variables. Regarding one of the variables as a dependent variable, the analysis employs a nonsymmetrical branching process, based on variance analysis techniques, to subdivide the sample into a series of subgroups which maximize one's ability to predict values of the dependent variable. Linearity and additivity assumptions inherent in conventional multiple regression techniques are not required. Some examples of its use are presented, as are formulas, accompanying research strategy and some unsolved problems. Indexing Descriptors: Computer Program, IBM 7090, Multivariate statistical analysis, Statistical interaction, analysis of survey data, prediction, analysis of variance, data analysis strategy, sequential decision procedures, simulation.

Section 1.2

## Introduction

This computer program (Identified as the (A)utomatic (I)nteraction (D) etector, Version 2) operates under the University of Michigan Executive System (1). It is focused on a particular kind of data-analysis problem, characteristic of many social science research situations, in which the purpose of the analysis involves more than the reporting of descriptive statistics, but may not necessarily involve the exact testing of specific hypotheses. In this type of situation the problem is often one of determining which of the variables, for which data have been collected, are related to the phenomenon in question, under what conditions, and through what intervening processes, with appropriate controls for spuriousness.

The data-model to which the present procedure is applicable may be termed a "sample survey model," in which values of a set of predictors $X_{1}, X_{2}, \ldots X_{p}$, and a dependent variable $Y$, have been obtained over a set of observations, or units of analysis, $U_{1}, U_{2}, \ldots U_{\infty} \ldots U_{n}$. A weight, $W_{\alpha}$, may also be established for $U_{\alpha}$ if sampling models are not representative and self-weighting are used, or if one observation is considered to be more reliable than another. Data may be considered "missing" or undefined on any of the $X_{i}$ or on $Y$. In particular, this analysis situation is defined to be one in which the $X_{i}$ are a mixture of nominal and/or ordinal scales (or coded intervals of an equalinterval scale) and $Y$ is a continuous, or equal-interval scale. The $X_{i}$ variables may consist of a mixture of "independent variables" and also "specifiers" (conditions) and "elaborators" (intervening variables). Thus, the problem is similar to the accounting or explanatory analysis described by Hyman (2).

The objective is to explain the variance of the dependent variable Y . Where the number of predictors is small, the problems of isolating the relationships between the $X_{i}$ and $Y$ are manageable, but when the number of predictors is large, which is typical of many survey data analysis problems, then an alysis of the joint effects of the $X_{i}$ on $Y$ presents serious problems. Many of these have been extensively discussed on the methodological literature. One summary is presented in

Morgan and Sonquist (3). Tukey (4) presents a searching critique of present data analysis techniques.

Data-analysis problems are translated into a variety of statistical questions. For instance, multiple regression techniques and other statistical procedures based on them attempt to answer the questions, "What is the effect of predictor variable $X_{i}$ on the dependent variable, holding 'constant' or removing the linear effects of the other predictors?" and "Are these effects 'significant' after taking into account the intercorrelations of the predictors?" The objective in an explanatory analysis is to ascribe the correct amount of the explained variation in $Y$ to each predictor, within the limitations of the linear and additive assumptions of the model, using least squares criteria. Thus, one way of handling the problem of determining the joint effects of a large number of predictors is to introduce linearity and absence-ofinteraction assumptions and then ask the above questions. The problem : is that in view of the present state of much theory, one typically doesn't know in advance which transformations (e.g., $X_{i}^{2}$ ) or interaction terms (e.g., $X_{i} X_{k}$ ) to introduce into the regression model, in order to produce a multi-dimensional surface over which the residuals are not on 1y normally distributed, but in which extreme values of the residuals. are scattered randomly over the surface [Ezekiel and Fox (5)]

A great deal of work has been done in several fields which are related to the problem focussed upon here. Belson (6) has suggested a sequential, nonsymmetrical division of the sample for the purpose of matching two groups on various characteristics used as controls in order to compare them. Tanimoto and Loomis (7) have developed a computer program which forms clusters of observations which are similar along a number of dimensions. Reiter (8) presents a stochastic algorithm for optimizing payoff functions. Alexander and Manheim (9) have developed a computer program for the analysis of correlational data. The intercorrelations between variables are represented as lines on a linear graph, which is broken into components using a "hill-climbing" algorithm based on the information-transfer between variables.

There are also studies going on in the selection of test items to get the best prediction with a limited set of predictors (10), usually using multiple regression. Westervelt (11) has developed an interesting approach to the problem of maximizing predictability with a minimum number of terms by using a step-regression model combined with artificial intelligence.

Group-screening methods have been suggested by Watson (12) and by Box (13) in which a set of factors is lumped and tested and the individual components checked only if the group seems to have an effect. These procedures have some similarity to the sequential process suggested here.

Our approach bears some resemblance to a formal decision procedure proposed by Duncan, Ohlin, Reiss and Stanton (14), using cost-utility curves and also to a sequential procedure suggested and tried by Danière and Gilboy (15). Earlier related work has been done by Wright (16) and by Kitagawa (17). Kretschmer and Vinton (18) have programed an "Information-Theoretic Seive" procedure which partitions a sample universe into two or more segments which are mutually exclusive and which minimize conditional uncertainty.

Each of these analysis schemes represents a specific statistical question. One such question is, "Given the units of analysis under consideration, what single predictor variable will give us a maximum improvement in our ability to predict values of the dependent variable?' This question, embedded in an iterative scheme is the basis for the algorithm used in this program. See (3, 19) for an extensive discussion of the rationale behind its development and implementation. The program divides the sample, through a series of binary splits, into a mutually exclusive series of subgroups. Every observation is a member of exactly one of these subgroups. They are chosen so that at each step in the procedure, their means account for more of the total sum of squares (reduce the predictive error) than the means of any other equal member of subgroups. The procedure may be described as follows.

Section 1.3 Description of the Algorithm

1. The total input sample is considered the first (and indeed only) group at the start.
2. Select that unsplit sample group, group i, which has the largest total sum of squares

$$
\begin{equation*}
\mathrm{TSS}_{i}=\sum_{\alpha=1}^{N_{i}} Y^{2}-\frac{\left(\sum_{\alpha=1}^{N_{i}} Y_{\alpha}\right)^{2}}{N_{i}} \tag{1.3.1}
\end{equation*}
$$

such that for the i'th group

$$
\begin{equation*}
\mathrm{TSS}_{i} \geq R\left(\mathrm{TSS}_{\mathrm{T}}\right) \text { and } N_{i} \geq M \tag{1.3.2}
\end{equation*}
$$

where $R$ is an arbitrary parameter (normally . $01 \leq R \leq .10$ ) and $M$ is an arbitrary integer (normally $20 \leq s \leq 40$ ).

The requirement (1.3.2) is made to prevent groups with little variation in them, or small numbers of observations, or both, from being split. That group with the largest total sum of squares (around its own mean) is selected, provided that this quantity is larger than a specified fraction of the original total sum of squares (around the grand mean), and that this group contains more than some minimum number of cases (so that any further splits will be credible and have some sampling stability as well as reducing the error variance in the sample).
3. Find the division of the $C_{k}$ classes of any single predictor $X_{k}$ such that combining classes to form the partition $p$ of this group i into two nonoverlapping subgroups on this basis provides the largest reduction in the unexplained sum of squares. Thus, choose a partition so as to maximize the expression

$$
\begin{equation*}
\left(n_{1} \overline{\mathrm{y}}_{1}^{2}+\mathrm{n}_{2} \overline{\mathrm{y}}_{2}^{2}\right)-\mathrm{N}_{\mathrm{i}} \overline{\mathrm{Y}}_{\mathrm{i}}^{2}=\mathrm{BSS}_{\mathrm{ikp}} \tag{1.2.3}
\end{equation*}
$$

where $N_{i}=n_{1}+n_{2}$
and $\quad \bar{Y}_{i}=\frac{n_{1} \bar{y}_{1}+n_{2} \bar{y}_{2}}{N_{i}}$
for group $i$ over all possible binary splits on all predictors, with restrictions that (a) the classes of each predictor are ordered into descending sequence, using their means as a key and (b) observations belonging to classes which are not contiguous (after sorting) are not placed together in one of the new groups to be formed. Restriction (a) may be removed, by option, for any predictor $X_{k}$.
4. For a partition $p$ on variable $k$ over group $i$ to take place after the completion of step 3 , it is required that

$$
\begin{equation*}
\mathrm{BSS}_{i k p} \geq \mathrm{Q}\left(\mathrm{TSS}_{\mathrm{T}}\right) \tag{1.3.4}
\end{equation*}
$$

where $Q$ is an arbitrary parameter in the range $.001 \leq Q<R$, and $T_{T S}$ is the total sum of squares for the input sample. Otherwise group i is not capable of being split; that is, no variable is "useful" in reducing the predictive error in this group. The next most promising group (TSS ${ }_{j}=$ maximum) is selected via step 2 and step 3 is then applied to it, etc.
5. If there are no more unsplit groups such that requirement (1.3.2) is met, or if, for those groups meeting it, requirement (1.3.4) is not met (i.e., there is no "useful" predictor), or if the number of currently unsplit groups exceeds a specified input parameter, the process terminates.

The following results, contrived, but realistic, will illustrate the basic output of the procedure. Suppose that Age, Race, Education, Occupation, and Length of Time in Present Job, are used in an analysis to predict Income. Age is an ordered series of categories represented by the numbers [1,2, ..., 6]. Race is coded [1 or 2], Occupation is coded $[1,2, \ldots, 5]$, Education is coded $[1,2,3]$, and Time on Job is coded [1,2, ..., 5]. We find the following mutually exclusive groups whose means may be used to predict the income of observations falling into that group:

| Group | Type | N | Mean Income | $\sigma$ |
| :---: | :---: | :---: | :---: | :---: |
| 12 | Age 46-65, white, college | 8 | \$8777 | \$773 |
| 13 | Age under 45, white, college | 12 | 6005 | 812 |
| 10 | Age 36-65, white, no college, nonlaborer | 24 | 5794 | 487 |
| 11 | Age under 35, white, no college, nonlaborer | 16 | 3752 | 559 |
| 9 | Age under 65, white, no college, laborer | 10 | 2750 | 250 |
| 5 | Age under 65, nonwhite | 10 | 2010 | 10 |
| 3 | Age over 65 | 10 | 1005 | 5 |
| Total |  | 90 | 4434 | 2263 |

A one-way analysis of variance over these seven groups would account for 95 per cent of the variation in income.

These results are arrived at by the following procedure, as represented by the tree of binary splits:


When the total sample (group 1) is examined, the maximum reduction in the unexplained sum of squares is obtained by splitting the sample into two new groups, "age under 65" (classes $1-5$ on age) and "age 65 and ovex" (those coded 6 on age). Note that each group may contain some nonwhites and varying education and occupation groups. Group 2, the "under-65" people are then split into "white" and "nonwhite." Note that group 5, the "nonwhites" are all under age 65. Similarly the "white, under age 65" group is further divided, into college and noncollege individuals, etc. A group which can no longer be split is marked with an asterisk and constitutes one of the above final groups. The variable "Length of Time in Present Job" has not been used. At each step there existed another variable which proved more useful in explaining the variance remaining in that particular group.

The predicted value $Y_{\alpha}$ for any individual for any individual $\alpha$ is the mean, $\bar{Y}_{i}$, of his final group. Thus $Y=\bar{Y}_{i}+\varepsilon$, where $\varepsilon$ is an error term. Prediction of income on the basis of age, education, occupation and race would provide a considerable reduction in error. Variables which "work" are, of course, the most logical candidates for inclusion in a theoretical framework.

We now turn to a description of the computer program, its organization, and use.

## CHAPTER II

## USING THE PROGRAM

## Section $2.1 \quad$ Program Organization

The program is written in MAD (Michigan Algorithm Decoder), a compiler language developed by Galler, Arden and Graham (20) for the IBM 704, 709 and 7090 systems. It uses several subroutines written in UMAP (University of Michigan Assembly Program), which is a modification of the standard assembly programs available through the IBM user's organization SHARE. MAD and UMAP are contained in the University of Michigan Executive System (1). Loading the program, program segmentation, input and output, and the need for numerous subroutines contained in the System require that AID (2) be operated in the context of the U. of M. System. The System, MAD, and UMAP are available through the IBM user's organization, SHARE. The program requires a 32 k system with 8 tape units.

AID (2) is organized into three program segments, the Editor or control segment, the Iterator or processing segment, and the Final Output Segment. Control originates in the Editor, is passed to the Iterator, then to the Output Segment and is then returned to the Editor, or to any program segment which may precede it on the program segment tape.

The functions of the Editor are to:

1) Read in control cards which describe
a) the location of the input data (tape or cards) and where it is to be stored.
b) which variables are to be used in the analysis and what they are to be used for.
c) what subset of the input data is to be used in the analysis.
d) other aspects of the current problem.
2) Read in the data and store it on tape if necessary.
3) Store the data to be used in the analysis into the appropriate positions of core storage.
4) Compute various statistics needed by the Iterator.

If errors occur, such as control cards out of sequence, problem too big for the program, illegal data, etc., the Editor provides appropriate diagnostic comments and then exits to the U. M. Executive System monitor.

The Iterator performs the analysis indicated by the parameters on the data provided for it by the Editor and provides intermediate output as requested. Threaded lists (21) are employed in the algorithm implementing the partitioning process.

The final output segment then calculates various statistics and prints out a summary of the results. It also calculates predicted values of the dependent variable and residuals for each unit of analysis. It then returns control of the computer to the Editor, (or to any other program segment which the user desires to place in front of the Editor) .

The tapes used are listed below.

| Tape Number | Function |
| :---: | :--- |
| 1 | U. M. Executive System Tape |
| 2 | AID Program Segment Tape |
| 3 | Scratch Tape--used by AID |
| 4 | Scratch Tape--used by AID |
| 5 | Not used |
| 6 | Output Tape |
| 7 | Input Tape |
| 8 | U. M. Executive System Tape |

Since a large number of variables may be read in and stored on a scratch tape, several analyses may be performed in succession. An attempt has been made to provide considerable flexibility with respect to data formats, multi-stage analyses using residuals as the dependent variable, and selection of subsets of the input data for analysis.

Section 2.2 Data Input Requirements
It is assumed that the data have been punched on IBM cards; one or more cards per observation. Input data may be punched anywhere on the card except in column l. Column 1 of the data cards may contain any legal character except an alphanumeric E. A legal character is defined as any punching pattern obtainable from a single depression of a key on a keypunch.

Since several analyses may be performed during one machine run, it is desirable to list the types of variables that may be entered into the computer. Each analysis may use its own subset of the variables. Variables entered into the computer are of five types:

1) Identifiers
2) Sample subset selectors (filters)
3) Predictors
4) Dependent variables
5) Weights

With the exception of identifiers, any variable may be used for purposes two through five above, provided it meets the restrictions made by the program on the values that variable may legally assume.

There are no restrictions on where any of the variables may be placed on the data cards, except that no variable to be used in an analysis may be punched in column 1.

Since the card reading equipment associated with the IBM 7090 operates in BCD mode, no data cards may be used by the program which have punching patterns anywhere on the card that do not constitute legal IBM characters.

The input data may be any file of (match-merged) data cards conforming to the above rules which can be described by nine cards of MAD format information. The MAD format information describes one unit $\mathrm{U}_{\alpha}$ of data.

All input variables except those which are to be used as identifiers are supplied to the program in Integer mode. Variables which are to be used as identifiers must be supplied to the program in Character (BCD) mode. Hence they may be used only for that purpose.

Thus, for any purpose except that of observation (unit) identifier, variables must be punched on the data cards in such a way as to permit their representation inside the computer as integers. Consequently, classes of predictors may not be represented as alphanumeric characters. However, there are certain special cases in which the characters + and - may be represented in the computer as integers. These are described in Appendix G. In general, the user is advised to represent his data on the IBM cards using only the characters 0 through 9, with the exception of variables to be used as dependent variables which may be signed numbers.

When several analyses are to be performed on the same set of data, machine costs will be somewhat reduced if all the variables to be used in all of the analyses are read in at the time of the first analysis, saved on tape, and subsequent analyses performed using the data from tape.

If card output of residuals is not desired and if, in addition, the analyses are to be performed on a subset of the sample, it will, in general, be cheaper to sort out the unwanted observations before setting up the run. If, however, punched residuals are desired, it is recommended that the entire sample be entered into the computer and the unwanted observations screened out using the sample subset selector (which will be described below). When residuals are requested, it is generally advisable to punch them, even if subsequent analyses are made of them from tape, since it is not always possible to anticipate which additional variables should be used in subsequent analyses of the residuals.

Section $2.3 \quad$ Program Capacity
Though data may be stored on tape, in the interest of computing efficiency, all of the information for any particular analysis, including predictors, dependent variable, weights, etc., are kept in core storage. Thus, the following limits apply:

Maximum number of input variables $=100$
Maximum number of dependent variables for any one analysis $=1$

Maximum number of predictor variables for any one analysis $=36$
Maximum allowable number of groups into which the input observations may be split $=63$

Range of any predictor $0 \leq V_{p} \leq 63$
Range that may be legally taken on by the dependent variable before scaling -99999 $\leq \mathrm{V}_{\mathrm{y}} \leq 999999$
Range that may be legally taken on by the weight associated with any given observation $0<\mathrm{V}_{\mathrm{w}} \leq 9999$
Range that may be legally taken on by any variable used as a sample subset selector $-99999 \leq \mathrm{V}_{\mathrm{f}} \leq 999999$
Maximum number of merged input data decks $=$ no limit, except that they must be able to be described by the MAD format statement

Maximum number of cards in the MAD format statement $=9$
Minimum number of observations that must be contained in the i'th group if that group is to become a candidate for splitting $2 \leq N_{i} \leq 999$
Maximum number of input observations: limits are determined by single-precision accuracy of 7090 floating point computations. A six-digit dependent variable, weighted by a three-digit weight is probably not subject to serious rounding error in calculating the total sum of squares until the sample size exceeds 5000 . No exact rounding and truncation error analysis has been performed.

Storage requirements are such that for any problem to be entered into ALD, the amount of storage per observation (3-8 words) times the number of observations must be less than 20000. Maximum sample sizes for all possible numbers of predictors are listed below. Determine which category the problem falls into, based on the number of predictors. The second line gives the maximum sample size.

|  | Category |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of Predictors (NP) in any one analysis | 1-6 | 7-12 | 13-18 | 19-24 | 25-30 | 30-36 |
| Maximum permissible number of observations in analysis | 6666 | 5000 | 4000 | 3333 | 2857 | 2500 |

If a problem is too large either the number of input observations must be cut down or the number of predictors must be reduced enough to put it into a different category. For instance, a problem with twelve predictors and a given sample size takes up as much space as one with seven predictors and the same sample size since both fall into category (b).

Complete page one of the Run Specification form (see Section 2.7). Make sure the data cards to be used as input are free of illegal punching patterns and that match-merging has been properly accomplished. Normally, the input data deck sequence is matchmerged on interview or data-unit number. Cards may be sorted into subgroups for entry into the computer, but in this case, a complete AID run must be made on each group separately. An accurate count of the number of input observations (N) must be made, as AID will throw the job off the computer if the parameter N does not agree with the actual number of observations read in.

Complete page two of the Run Specification form, the datadescription. List all variables to be used in all of the AID analyses, including predictors, dependent variables, identifiers, filters, and weights, starting from the left side of the input data cards and working toward the right. Then number the variables, sequentially, starting with the integer 1 . There are no restrictions in AID as to where the predictors or dependent variable or weight, etc., must be located, except for the fact that no variable may be in column 1. Column 1 on the first card of each data set may not contain the character $E$. There are no other restrictions on data except the usual ones. That is, no multiple punches may occur in the columns which are to be used as predictors, filters, weights or dependent variables; and no illegal punching patterns may appear anywhere on the card. If + or - punches occur in the predictors or the dependent variable as other than a sign for the dependent variable, confer with an experienced programmer before proceeding further (see Appendix G).

List for each variable (a) a name (up to 12-characters), e.g., AGE, INSURANCE, etc., and (b) the columm numbers in which the variable is located. For all variables which are to be used as predictors, filters, or dependent variables, list all of the possible values that variable can legally have. NOTE: NO VARIABLE

TO BE USED AS A PREDIĆTOR MAY LEGALLY HAVE A VALUE LARGER THAN 63. Nor may any predictor have a negative value. Note values of the dependent variable that should be omitted as missing data. Use the intended usage colum to indicate the function (identifier, predictor, filter, etc.) that variable is to perform. Illegal values of input data will result in an automatic exit from AID and a memory dump.

AID has the capacity to omit observations that have certain specified values of the dependent variable. All observations having the dependent variable $\mathrm{V}_{\mathrm{y}}=-0$ are automatically omitted. All values larger than a certain specified value may be omitted. In addition, all observations equal to either of two other specified values may be omitted. These values should be indicated on the data-description form. Unless there is a great deal of missing data, it is desirable to leave such observations in the deck and have the computer throw them out of the analysis, rather than sorting them out beforehand.

The purpose of the analyst's recording this information on these forms is to inventory all necessary information about the run in one place to prevent the inadvertent forgetting of a necessary piece of it. If any of the variables have the characters + or - used for anything other than a sign for a dependent variable, one of the special input formats provided by subroutine IRFORM must be used. Confer with an experienced programmer.

Use the collected information on the Data Description forms to fill out the remainder of the Run Specification forms which establish control card punching and the input file sequence.

## Labe1 Card--Type 1 card

Column one of this label card must be punched with a one (1). Punch 78 characters of Alphanumeric run identification (anything you can punch with one depression of a key on a keypunch) in columns 2-79.

If card residuals are requested, place the research project identifier starting in column 2 of this card, followed by a deck identification number. The contents of columns 2-13 of this card will be punched into the card residual output from this analysis. All BCD characters are legal project and deck identifiers. The entire contents of cols. 2-79 will be printed on the output statistics. If this run is a subset of the sample, i.e., a partial data deck has been entered into the computer, this information should be punched on the label card somewhere after col. 13.

Main Parameter Card--Type 2 card
This card contains a series of parameters identifying the location of the input data (cards or tape) that is to be used on the analysis, the number of observations to be expected from this source and the number of input variables. The remainder of the card is a series of parameters that form a sentence. This sentence defines what subset of the input observations are to be used in the analysis. It will be referred to as an input subset selector or "filter." The subset selector has no effect on what is transmitted to tape, but only defines what observations are to be packed into core storage for the present analysis.

A description of the parameters, their permissible values and their purposes follows:

| Colunm | Name of Parameter | Remarks |
| :---: | :---: | :---: |
| 1 | Card type | Must be punched with a 2. |
| 7 | LOCDAT | C if input data is on cards and is not to be saved on tape. |


includes the integers ( $-556,-555, \ldots .-1,+0,+1,2, \ldots 1088,1089$ ). Minus zero (-0) is specifically defined to be in this interval. The numbers -557 and 1090 lie outside the interval. (-556 to 1089) are considered to be inside the interval. In this example, the lower bound is defined as -556 and the upper bound is 1089.

A subscript or index of an input variable is that integer assigned to it when input variables are numbered from left to right across the (merged) data decks as described above. An input variable which is stored in the computer in BCD mode (identifiers are normally in this mode) may not be used in the sample subgroup selector.

The words AND and OR appear in the selector sentence. The two terms correspond to common English usage. Specifically, OR is inclusive, rather than exclusive. For completeness, they are described as follows:


The sentence contains a command (INCLUD, EXCLUD); a first condition, called condition A; a connector (AND, OR) ; and a second condition, called condition B. For example:

1) "INCLUDe in this analysis all input observations which are
2) OUTside the closed interval which runs from 1 (lower bound) up to 4 (upper bound) on the variable whose input number is 5
3) $O R$ which have values such that they are
4) INside the closed interval which runs from 2 (lower bound) up to 2 (upper bound) on the variable whose input number is 6."
(1) above is a command; (2) is condition " $A$ " which is either true or false for any given input observation $U_{\varnothing}$; (3) is a connector; and (4) is condition " B " which is either true or false for the observation $\mathrm{U}_{\alpha}$.

The above example specifies that if condition $A$ is true or if condition $B$ is true, or if both of them are true, then the observation will be included in the analysis. If both are false for that observation, then it will not be used.

Another way of stating this is to say that the conjunction of conditions $A$ and $B$ for any observation $U_{\alpha}$ is either true or false. If it is true, then the action specified by (1) is taken. If it is false, then the action complementary to that specified in (1) is taken. The actions which may be specified are INCLUDe or EXCLUDe. They are complementary.

It may be desired to establish only one condition for entry of an observation into the analysis. In this case the connector is left blank and the program ignores the parameters referring to condition $B$. Then, if condition $A$ is true for observation $U_{\alpha}$, the action specified in the command is taken. If condition $A$ is false, then the complement of the action specified in the command is taken.

It may be desired to use all of the input observations in the analysis. In this case, the command itself is left blank and all observations will be used, bypassing the subgroup selection process completely.

The exact description of the filter (input subset) parameters is given in a Section 2.7 entitled "AID (2) Run Specifications, Input File Assembly."

It should be noted that several other conditions will cause an observation to be excluded from an analysis. These conditions involve values of the dependent variable which are declared to be "missing data," and will be described later. These conditions operate independently of the sample subset selector.

Input variables which are to be used as identifiers for punched residual output may not be used as "filter" variables in the sample subgroup selector, since they are BCD in mode. However, if it is desired to perform an analysis on a subset of the input data definable in terms of the observation identifier, and if, in addition, matchmerged data decks are used as input, one of the unit identifier fields may be read in as an integer and used for a filter variable provided
it contains only the characters zero through nine in the field. Another identifier may then be stored in BCD (Character mode) and used for identifying the residuals.

Secondary Parameter Card--Type 3 card
This card contains the remaining parameters describing the analysis to be performed, with the exception of the list of variables that are to be used for predictors.


20-25 P2 $=$ The best split on the $i^{\prime}$ th candidate group must reduce the unexplained sum of squares by P2 proportion of the total sum of squares or that

| Colurn | Name of Parameter | Remarks |
| :---: | :---: | :---: |
|  |  | group will not be split, and it will not become a candidate group again even though it may meet the P1 requirement above. The range is: $\mathrm{P} 1 \leq \mathrm{P} 2 \leq$ .99999. The decimal point is punched in this field as above. The proportion .0060 has been found to work well with samples of 1500-3000. Other values may be punched at the user's option. Increase P2 to at least . 01 with sample sizes of $200-300$ or less. |
| 26-28 | MAXGP | $=$ The maximum allowable number of final groups into which the input data may be split, regardless of P1 or P2. Thus, the splitting process will always stop when the sample has been divided into MAXGP number of unsplit subgroups. MAXGP may not be larger than 63 , i.e., 62 splits, 125 groups in all. Fifty has been found to be a satisfactory maximum number of splits. The range is: $1 \leq \operatorname{MAXGP} \leq 63$. |
| 29-31 | MSIZE | $=$ This is the minimum number of observations that must be contained in a group if that group is to become a candidate for splitting. Its purpose is to prevent small groups with somewhat unstable means from being further split, since the splits are likely to be heavily influenced by sampling errors. Normally MSIZE should not be smaller than 25 . Range: $001 \leq$ MSIZE $\leq 999$. |
| 35-37 | Y | $=$ This is the index number $f$ the variable to be used as the dependent variable. For example, if the dependent variable is the 14th variable on your Input Description form, then punch 014 here. If the dependent variable is number four, punch 004. NO VARIABLE TO BE USED AS A DEPENDENT VARIABLE MAY HAVE A VALUE LARGER THAN 999,999 or less than -99999 before scaling. |


| Column | Name of Parameter | Remarks |
| :---: | :---: | :---: |
| 38-49 | YNAME | $=$ Punch alphanumeric information here. The name of the dependent variable, e.g., INCOME, WIFE'S WAGE, etc. |
| 50-55 | YMAX | $=$ This is a "missing-data" code. For some observations, there may be no information on the dependent variable. Or there may be large values which are to be screened out. These may be left in the computer input file. YMAX is for preventing them from being used in the analysis. Thus, any observation whose dependent variable has a value algebraically larger than YMAX will be read, but not used by the computer in this analysis. YMAX is scaled by the input scale factor before being used. If you do not wish to use YMAX, leave it blank. |
| 56-61 | MDI | $=$ This is an additional method of throwing missing data out of the analysis. Any observation such that the dependent variable is exactly equal to MDI will not be used in the analysis. MDl is scaled by the input scale factor before being used. If you do not wish to use MD1, leave it blank. |
| 62-67 | MD2 | $=$ The same as MD1. Do not use MD2 without using <br> MD1 also. Leave it blank if you do not use it. |
|  |  | Note on missing data: regardless of what is punched in YMAX, MD1 and MD2, AID will omit all observations such that the dependent variable |
|  |  | has the value minus zero. If all of your missing data are coded in this fashion, or if you have no missing data, then leave YMAX, MD1, and MD2 blank. All undefined residuals have the value -0. |


| Column | Name of Parameter | Remarks |
| :---: | :---: | :---: |
| 68-70 | CDRES | $=$ If it is desired to compute residuals for this analysis and punch them on cards, this parameter is punched CRD, otherwise it must be left blank. If residuals are to be punched on cards, columns 2-13 of the label card (type 1) must contain research project and deck number information. An identifier variable must be included as part of the set of input variables and must be made available to the program in BCD (character) mode. This variable must be indicated by a nonzero value for the parameter INTNO described below. |
| 71-73 | TPRES | $=$ If it is desired to compute residuals and write them on tape for a subsequent analysis, this parameter is punched TAP, otherwise it must be left blank. This option may be exercised regardless of whether the input data for this analysis is on cards or on tape. If it is exercised, then the residual is written on tape as variable NV +1 , where NV is defined as above. IF A SUBSEQUENT ANALYSIS IS TO BE PERFORMED ON TAPE, THE PARAMETER NV ON THE FOLLOWING ANALYSES MUST BE ADJUSTED ACCORDINGLY, as there is now one more input variable. |
| 74-76 | INTN0 | $=$ This is the index, or subscript of the input variable (identifier) to be punched in the interview number field of the output cards containing residuals. If card residuals are being obtained from this analysis, this parameter must lie in the range $1 \leq$ INTNO $\leq N V$. If card residuals are not being obtained from this analysis, then ININO may be left blank or set to zero. It will not be interrogated. |


| Colunm | Name of Parameter | Remarks |
| :---: | :---: | :---: |
| 77-78 | SCFIN | $=$ This is an input scale factor to be applied to Y , to YMAX and to MD1 and MD2. It is that power of ten by which $Y$ is to be multiplied before being used in computation. Thus, the characters 12345 read as a five-columm Integer (I) field on a data card, or from tape, will have the internal value of 12.345 , if this parameter has the value -3. The pur pose of this parameter is to determine where the decimal appears in the printed output. For analys.is of residuals, where a previous SCFOUT has moved the decimal point to carry more significant digits, SCFIN is used to put the decimal point back in the right place for this analysis stage. In this case SCFIN equals the previous SCFOUT with opposite sign. <br> Range: $-9 \leq$ SCFIN $\leq+9$. |
| 79-80 | SCFOUT | $=$ This is an output scale factor which is applied to $Y$, the predicted value of $Y$ and the output residual, after computation and before punching or the writing of the residual on tape takes place. It is that power of ten by which these terms are to be multiplied before being output as integers. It will generally be desirable to provide more significant digits in the residuals than there were in the original dependent variable. Therefore, SCFOUT is normally equal to $[(-S C F I N)+2]$, reducing the dependent variable to its original form and adding two more significant digits. Range: $-9 \leq$ SCFOUT $\leq+9$. The purpose of SCFOUT is to move the decimal point in the (previously scaled) dependent variable into a place suitable for punching or writing on tape. |

## Predictor List Cards

The user must supply information to AID telling it which of the input variables are to be used as predictors. (The information on the main parameter cards has indicated which input variables are to be used as the dependent variable and the weight, if desired.) Each predictor list card contains information on up to four predictors. The last predictor card is the only card that may contain information on less than four predictors. Any input variable may be used as a predictor provided it is stored in the computer in integer mode, never exceeds the value 63 and is never negative in value.

The predictors may be listed in any order desired by the user, since the order listed is irrelevant for the program. Three types of information are punched for each predictor: its index, a type code and its name. The index is obtained from the Data Description sheet. It is the field number established by numbering the NV variables from left to right across the merged input decks. The name of the variable should be punched as up to 12 characters representing a suitable memonic reference to the substantive meaning of the variable, e.g., AGE, SEX, INCOME, REGION; RISK SCALE, atc. A blank is counted as a character.

The predictor type is punched as $M$ (monotonic), or $F$ (free). Predictors identified as type " $M$ " will have the order of their coded values ( $0,1, \ldots, k, \ldots, 62,63$ ) maintained during the partition scan. In this case the classes of the predictor will not be re-arranged by sorting them into descending sequence using the mean value of Y for each class as a key. In designating a predictor, say $V_{p}$ a type $M$ predictor, the user assumes that though the function $Y=\bar{Y}_{k p}$ may not be linear it is at least monotonic. The usual use for a type M restriction is to apply it to an ordinal scale, or to class-interval codes established for a continuous variable with an expected monotonic effect on the dependent variable.

Predictors identified as type " F " will have their classes re-arranged during the partition scan. They will be sorted into descending sequence using the mean value of $Y$ for each class as a key.

The usual use for a type $F$ predictor classification is for variables that are nominal scales, or for other cases in which it is suspected that the function $Y=\bar{Y}_{k p}$, where $k$ is the predictor class code, is not monotonically increasing or decreasing. A useful strategy may be to classify all predictors as type $F$, determine whether partitions appear that look fortuitous, and then to restrict the offending predictor (s) in a subsequent analysis.

Punch only as many predictor cards as are needed. Up to 36 predictors may be used (nine predictor cards). Each card should be completely filled in, except the last one, which will have some blank spaces at the end if NP is not an exact multiple of four. The format of the predictor list cards is described hereafter in the AID (2) Run Specification, Input File Assembly.

## MAD Format Statement

The MAD format statement is punched in columns 2-72 inclusive, on up to nine (9) cards. IT MUST BE COMPLETELY ENCLOSED IN PARENTHESES, as it is read in by subroutine IRFORM. There must be exactly NV field descriptions, in addition to the
(C1,
that starts the format statement. Included are all predictor(s), the dependent variable(s), identifier(s), filter (s) and the weight(s) for all analyses to be performed, together with the appropriate $S$ (skip) and / (go to the next card) characters. All columns of the input data starting with column li of the first merged deck and continuing to the last (rightmost) variable of the last merged deck must be accounted for. The first colum on the first merged deck is accounted for by the (C1,
on the first card of the MAD format statement. The format statement ends with the characters
*)
A11 fields used for input must be specified in integer (I) mode, except for identifiers which are character (C) in mode. Insert only as many format cards as needed. See the MAD manual (20) for additional details.

See Appendix $G$ of this write-up for a description of Subroutine IRFORM which reads in the MAD Format information, especially if the data cards to be used contain other than the characters $0-9$ in the variables to be used as predictors or filter variables; or if the dependent variables contain punching patterns other than signed numbers or minus zeroes. An example follows for $\mathrm{NV}=7$ and one input deck:

$$
(C I, S 3,4 I 1, I 4, I 2, S 60, C 6 *)
$$

For two merged input decks and $\mathrm{NV}=7$ one might write:

$$
(\mathrm{C} 1, \mathrm{~S} 3,4 \mathrm{I} 1 / \mathrm{S} 8, \mathrm{I} 4, \mathrm{I} 2, \mathrm{~S} 60, \mathrm{C} 6 *)
$$

In the first example variables are located as follows:

| Index No. | Cols. |  | Function |
| :---: | :---: | :--- | :--- |
| 1 | 5 |  | Predictor |
| 2 | 6 |  | Predictor |
| 3 | 7 |  | Predictor |
| 4 | 8 |  | Predictor |
| 5 | $9-12$ |  | Dep. Var. Y |
| 6 | $13-14$ | Weight |  |
| 7 | $75-80$ |  | Identifier |

In the second example variables are located as follows:


## Section 2.6 Input File Assembly Sequence

An AID Run Requires the Following Input File
(1) Two computing center job cards (See U. of M. Executive System Write-Up [reference (1)] for a description)
(2) A systems card \$EXECUTE, DUMP, I/O DUMP, BINARY
(3) The AID program decks, in binary form
(4) A systems card \$DATA
(5) An AID label card (type 1 card)
(6) An AID main parameter card (type 2 card)
(7) An AID secondary parameter card (type 3 card)
(8) Up to nine (9) AID predictor list cards (type 4 cards). Insert as many as needed, no more
(9)* Up to nine (9) cards containing a MAD format statement enclosed in parentheses. Insert only as many cards as needed, no more.
(10)* A DATAFOLLOWS card
(11)* The match-merged data-decks
(12)* A Type E trailer packet
(13)* As many repetitions of (5) - (12) above as desired
*These cards are omitted if the data are already on tape from a previous analysis.

Section 2.7 AID (2) Run Specifications, Input File Assembly
These forms were developed as an aid to taking an inventory of all the information necessary to initiate a run on AID (2). Taken together, and properly completed, they provide the user with the source material necessary for keypunching his control cards and assembling his input file.


PLEASE INCLUDE COMPLETE IDENTIFICATION AND CARD COUNT OF ALL DECKS USED. INDICATE WHICH COLUMNS CONTAIN THE STUDY NUMBER, DECK NUMBER, AND INTERVIEN NUMBER.
COMPUTER PROGRAMS(s) TO BE USED: (A) utomatic (I)nteraction (D)etector (Model 2) PREREQUISITES:

Purpose: (description of dependent variables, predictors, whether multi-stage run, etc.)

Number of file assembly packets (pages 3-8) included in this run $=\square$


|  | Number: | in columns: | Number: | in columns: |
| :--- | :--- | :--- | :--- | :--- |

Sight check identification and verify all $N^{\prime}$ s on sorter before proceeding further.
Special Instructions: (match-merging of decks, cards to be omitted from computer input file, request for checking of invalid punching, etc.)

Number of observations in computer input file $=$
(Control card 2, col. 8-13)
NOTE: Prior to any 7090 run, all decks should be checked for blank columns and double punches. If this has not been done previously, request deck checks as a preliminary step in this request.

AID(2) Run Specifications, Input File Assembly
Computer Input Data Description
$\operatorname{AID}(2)$

| $\begin{gathered} \text { FIELD } \\ \text { NO. } \\ (1) \end{gathered}$ | VARIABLE NAME (2) | $\begin{gathered} \text { IN } \\ \text { COLS. } \\ (3) \end{gathered}$ | MAD FORMAT* <br> (4) | $\begin{gathered} \text { POSSIBLE } \\ \text { CODE VALUES } \\ \text { (5) } \end{gathered}$ | INTENDED USACE <br> (6) |
| :---: | :---: | :---: | :---: | :---: | :---: |
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*Filled in by programmer April 1, 1964
(Insert extra pages as necessary)
Page of

AID(2) Run Snecifications, Input File Assembly

> (A) utomatic (I) nteraction (D)etector
> MODEL 2
> File Assembly packet
> $(1$ APR 64)
I. Label Card: Type 1 innut parameter card, Column 1 Must be punched with a 1 .

(78 Characters of Alohanumeric Run Identification. Eligible characters include $0-9, A-Z, S+-/ *=$, ( ) and blanks. If punched residuals are requested, then columns $2-13$ of this card will be punched into cols. 1-12 of all residual card output for this run. Thus, alphanumeric study and deck information can be transferred to output.) Cols. 2-79 of the label card are always printed on the output.
i Uncil all parameters right-adjusted in the field on all following parameter CARDS
II. Type 2 parameter card:


INPUT SAMPLE SUB-GROUP SELECTOR (Filter)
These parameters define (if desired) a subset of the input observations which are either to be included or excluded from this run. Cross out filled in options which do not apply.


In this run all input observations which are
(lover bound), up to

(upper bound), on the variable whose input number is

(subscript of filter variable),

which have values such that they are

(lower bound), up to

(upper bound), on the variable whose input number is (subscript of filter variable).

Leave blank


Note: If cols. 20-25 are blank, then all input observations will be used in the run and the parameter cols. 26-79 are ignored. If cols. 50-55 are blank, then the parameters in cols. 56-79 are ignored. The above sample sub-group selection does not affect which observations are written on tape (LOCDAT $=W$ ). It only determines which observations will be allowed to enter this analysis. DO NOT USE COLS. 56-79 UNLESS YOU ALSO USE COLS. 20-55. Do not use interview number as a filter variable.

AID(2) Run Specifications, Input File Assembly

## III. Parameter Card Type 3



$=$ The best split on the ith candidate group must reduce the unexplained sum of squares by P2 proportion of the total sum of squares, or that group will not be split and will not become a candidate group again. The decimal point is punched in the field, e.g., . 006 (split reducibility criterion).

$=$ MAXGP $\quad=$ The maximum allowable number of final groups into which the input data may be split, regardless of P 1 or P 2 . (002 $\leqq$ MAXGP $\leqq 063$ ) . Normal $=050$.

$=$ MSIZE $=$ Minimum number of observations that must be contained in the ith group if that group is to become a candidate for splitting. Normally MSIZE $=25$.



AID(2) Run Specifications, Inp̣ut File Assembly

Col.

$=$ CDRES $=$ If it is desired to punch residuals from this analysis on cards, this parameter is punched CRD, otherwise it must be left blank.
$=$ TPRES $=$ If it is desired to compute residuals and urite them on tape for a subsequent analysis, this parameter is punched 'TAP, otherwise, it must be left blank.
(Note: If this option is exercised, then the residual is written on tape as variaile iV+l. AND THE PARA:LITIR NV ON THE FOLLOWTNG ANALYSIS HUST BE NDJUSTED ACCORIIINGLY, as there is no: one more innut variable.)
$=$ InTwo = Index of the input variable to be punched in the Interview iJumber field of output cards containing residuals. This field iUST lie in the range $1 \leqslant \operatorname{INT}(\mathrm{No} \leqslant \mathrm{NV}$ whenever residuals are to be punched, otherwise it may be left blank or set to zero.

$=$ SCFLN = Input scale factor. This is that power of ten by which $Y$ is to be multiplied before being used in computation. Use 0 for one-zero "dummy" variable dependent variailes. Use -4 for residuals of such dummies. Range ( $-9 \leqslant$ SCFLN $\leqslant+9$ ). For residuals, normally SCFI:S $=$ the previous SCFOUT with the opposide sign.

$=\mathrm{MD} 2$
$=\mathrm{KD1}$
$=$ Any observation such that $V_{y \alpha}=$ MDl will not be used in the analysis. Note: The program also automatically bypasses all observations such that $V_{y x}=-0$. (-99999 $\left.\leqq \mathrm{MDI} \leqq 999999\right)$ inote 2: Leave MDl blank if not to be used.
$=$ Any observation $\alpha$ such that $V_{y \alpha}=M D 2$ will not be used in the analysis. (-99999 MD2 $\leqq 999999$ ). Leave IID2 blank if not to be used.

$=$ SCFOUT $=$ Output scale factor. This is that power of ten by which Y, predicted $Y$ and the residual are to be multiplied before the punching or writing of the residual on tape takes place. Range ( $-9 \leqslant$ SCFOUT $\leqslant+9$ ). Use +4 for one-zero "dummy" variable dependent variables. Use +4 for residuals of such dumnies. Normally SCFOUT $=$ [ (-SCFIN) +2 ]
IV. Type 4 Parameter cards: Predictor list cards.

Column 1 MUST be punched with a 4, on all predictor cards. Insert only as many predictor list cards as are needed to account for NP predictors. Values of all variables used as predictors must lie in the range $0<X_{i} \leqslant 63$. The maximum number of predictors is 36 . Do not leave any blank fields on the predictor cards except on the physically last card after the last predictor. The Variable Number is the input field number. The predictor type is punched $M$ if the predictor code ordering for that predictor is to be maintained during the partition scan. If the categories of that predictor are to be sorted into descending sequence on their means, then the predictor is a type $F$ (Free) predictor, and the parameter $\mathrm{T}_{\mathrm{p}}$ is punched F . Names may include all eligible characters listed on page one.


AID (2) Run Specifications, Input File Assembly
this page to be filled in by programmer
IV. IHAD Format Statement:* Up to 9 cards, cols. 2-72.

Cols. ENCLOSE IIV PARENTHESES


There must be exactly $N V$ field descriptions, including all predictors, the dependant variable and the weight (if the latter is to be used), together with appropriate $S$ (skip) and / (go to next card) characters. If residuals are to be punched, either on this run, or on any subsequent runs using tape input, then the NV field descriptions MUST include an observation identification field (normally the interview or subject number). This is a C (character) field in mode. Column 2 of the first data card through the units position of the rightmost variable on the last (merged) deck must all be accounted for. All fields are integer (I) in mode except for the interview number field, which is a C (character) field. Insert only as many format cards as are needed. Note the first MUST start with ( Cl , and the last MUST end with *).
v. DATAFOLLOWS Card:*

Cols.

| 11 | 12 |
| :--- | :--- |
| DATAFOLLOWS |  |

VI. Insert (match-merged) data deck (s) here.* The number of observations must agree with the parameter $N$. The number of cards is $D(N)$, where $D$ is the number of merged decks. $D \neq 0$.
VII. Type E Data Trader Packet:* Insert a packet of $D$ cards, where $D$ is the number of (merged) input decks. An $E$ is punched in column l of the first card. The remaining cards are blank.
VIII. As many repetitions of $I$ through VII as desired.

## Section $2.8 \quad$ Program Timing Estimates

Timing examples:


Section $2.9 \quad$ Output Page Estimates $P=4+Q+(M \times Q)+\frac{3 M}{2}+\frac{M}{2}$
where $Q=\frac{\sum_{x=1}^{N P} C_{x}+5 N P}{25}$
$\mathrm{M}=$ maximum allowable groups
$\Sigma C_{x}=$ the sum of $C_{x}$ over all predictors where $C_{x}$ is the largest (numeric value) code in the predictor $x$ $N P=$ the number of predictors

Section $2.10 \quad$ Printed Output Available from the Program

For each analysis:

1. An identifying label.
2. Number of input observations, number of input variables, number of predictors, index of the variable used as a weight, group spliteligibility criterion, group split reducibility criterion, minimum group size, maximum number of allowable groups, index and name of the dependent variable, a definition of missing-data values of the dependent variable which were used in deleting such observations, decimal point locators (scale factors), location of input data, and a definition of which subset, if any, of the input observations were specified for use in the analysis.
3. A dictionary of where the variables came from on the data cards and a record of the mode in which they were stored in the computer, and program timing information.
4. A listing of all of the predictors used in the analysis, their maximum values and the type of predictor (free or monotonic).
5. Statistics for the total number of observations in the analysis including total read, total deleted, total used, and, for the latter, the total sum of weights, sum and sum of squares of the dependent variable, its mean and standard deviation, the total sum of squares (TSS) for the analysis, and the two values PA and PB, that is, the sum of squares that must be contained in a group if it is to be split, and the sum of squares that must be transferred from within to between-group sums of squares for a split to take place.
6. a. A record of the statistics for all attempted partitions of the entire sample (group 1), over all classes of all predictors.
b. These statistics include, for each class, the number of observations, the sum of the weights, $\Sigma \mathrm{Y}, \Sigma \mathrm{Y}^{2}, \bar{Y}, \sigma$, and the $B S S_{i k p}$ for each possible partition between adjacent classes, and the total sum of squares in the group under attempted partitioning.

Final output for each analysis consists of:

1. A complete definition of each group created during the partitioning process, including the group identification number, the identification of the 'parent' group from which it was split, identification of the variable used to split off this group, the classes of the partitioning variable forming the group, and an indication whether the group was retained as a final group; for the group, the statistics $N, \Sigma w, \bar{Y}, \Sigma Y, \sigma, \Sigma Y^{2}$, deviation of the group mean from the grand mean, weighted proportion of the total observations used which are in the group, weighted mean square for the group, the proportion of the total sum of squares in the group, and the sum of squares for the group.
2. A one-way analysis of variance table over the final groups. This should be interpreted with extreme caution, especially when weighted data are used.
3. By option, residuals (discrepancies between observed and predicted values of the dependent variable) may be punched or written on tape, or both, for subsequent analysis. Punched output includes identifying information supplied by the user, the observation number, the identification number of the AID final group into which the observation fell, the predicted value for the observation, its actual value on the dependent variable, and the residual score. Scale factors punched on the control cards provide for the desired number of significant digits in the residuals.

Section 2.11 Residual Output on Punched Cards
If residuals are requested in punched card form, the following output will result for each analysis. One card will be punched for each observation initially read into the computer whether it was used in the analysis or not. These cards have the following format. (Note that the dependent variable is read into the computer as an integer.)

| Columms | Content | Remarks |
| :---: | :---: | :---: |
| 1-12 | Identifying information | Obtained from cols. 2-13 of control card 1 |
| 13-18 | Observation number | Obtained from the input variable identified as the observation number on control card 3. This is a BCD character (C) field and is punched left-justified in the field established for it in the output card. |
| 19-21 | Group number | The identification number of the final group of which this observation is a member. If the observation was not used in the analysis, this is zero. |
| 22-29 | Predicted <br> Value of $Y$ | This is the mean of the final group of which the observation is a member. If the observation was not used, then this has the value -0 . When present this quantity is obtained by computing the group mean to 8 -place floating point accuracy, multiplying the result by the output scale factor (decimal point locator) and then rounding it to the nearest integer for punching. (Input values of the dependent variable must be integer in mode, but may be scaled appropriately via a control card parameter. |
| 30-35 | Actual Value of the Dependent Variable Y | Obtained from the input variable designated as $Y$ by the parameter on control card 3 . |


| Colurm | Content | Remarks |
| :---: | :---: | :---: |
| 36-43 | Residual | $\mathrm{R}=\mathrm{Y}-$ <br> final gr <br> member <br> multipli <br> point lo <br> for punc <br> in the |
| 44-50 | Weight | This is <br> Otherwi <br> ble des |
| Note: | Normally the should conta followed imm deck produce exactly the For group me residuals, nonzero digi blank. | ents of e resear ely by th the comp order as values of are punch the rema |

Section $2.12 \quad$ Residual Output on Tape
If residuals are requested for storage on tape, they will be computed as indicated above and then stored on a data-tape along with all variables entered as input. Thus, if the input consists of NV variables, predictor variables, dependent variables, weights, filters, etc., then the residuals will be written on the tape as variable NV +1 . Residuals which are undefined, either because the dependent variable is undefined (missing data) for that observation or because the observation was prevented from being used in the analysis by the use of the "filter" will have a value of -0 . They will be omitted automatically from any subsequent analysis which uses the data on the tape and which specifies these residuals as the dependent variable.

Each analysis specifying residuals to be left on tape will result in additional variable; the residual from that analysis will be left on the tape. Thus, suppose four analyses are performed. The first specifies card output of residuals. The data input consists of 56 variables. It is requested that the data be saved on tape. At the end of this analysis, there will be 56 variables on tape. The control cards for the following analysis will specify 56 input variables. If they specify residuals to be left on tape, then, at the end of that analysis, the tape will have 57 variables on it, the 57 th being the residuals requested. A further analysis using the tape must specify 57 variables as input. If tape residuals are again requested, then after the termination of this analysis, there will be 58 variables on the tape. The fourth analysis, if it is to use the tape, must specify 58 variables as input. A fifth analysis, if the data come from cards, may either write a new tape or ignore it, but may not add additional residuals to it. Thus, any time data come in from cards and either tape residuals or the saving of the data on tape are requested, a new tape is written and the old one destroyed. There are no provisions in the program for saving tapes which have been written. It is assumed that the primary datastorage mode is on cards.

## ILLUSTRATIONS AND EXAMPLES

## Introduction

We present a series of thumbnail analyses drawn from computer runs that were made on the program. Our objective is to illustrate the output available from the program, analysis strategy with respect to its interpretation, and to point out the sensitivity that the method has when problems occur, such as a skewed dependent variable or un-interpretable splits associated with predictors of considerable conceptual complexity.

A number of the trees presented use sets of predictors that had previously been employed in a multiple classification analysis. This technique (22) is equivalent to a dummy-variable multiple regression (23). One objective has been to determine whether the findings based on the trees were consistent with previous analyses, and whether additional information about the structure of relationships between the variables could be extracted from the trees. With a few exceptions, which will be noted later, these expectations appear to be fulfilled.

Nine examples are presented. The first is a two-stage analysis where the objective is a stringent test of the effectiveness of a factor (occupation) known to have a very powerful effect on average hourly earnings. Complete documentation of the entire run is presented, including a listing of the input, codes for the variables and the computer output.

The second example (home ownership) illustrates the use of a dependent variable which is dichotomous, rather than equal-interval. Parsimonious explanation is achieved, together with clear evidence that neither family size nor age are uniform in their effects throughout the population.

The following example (plans to move) introduces an assumption of an underlying continuum. The concept of alternative inhibiting factors is illustrated. The fourth example (nonfamily contributions) illustrates the type of analysis probiems that arise when the dependent variable is badly skewed. An analysis strategy for handling this problem is presented. The effects of using predictors, which are themselves complex indices representing several dimensions, are illustrated. Several questions which the analyst should raise when interpreting the tree output are suggested.

The next example (expected family size) constitutes a re-analysis of data which have been extensively studied, to determine whether the behavior of the variables in the trees were consistent with previous findings. Generally, this was found to be the case. However, the importance of keeping the number of classes in the predictors to a minimum and of constraining the ordering of those which have a natural ordering to them is clear. The illustration emphasizes the need for predictors which are as uni-dimensional as is possible. The sensitivity of the procedure to this type of conceptualization problem indicates its possible use in locating concepts in need of refinement. Coding the offending variable somewhat differently may then be possible, leading to better discriminatory power for it when used.

The following example (average completed education) illustrates the use of several methods of displaying the results for further analysis, together with a hypothesis suggested by one of the splits.

The seventh example (disposable income) illustrates a nonsymmetric effect by a series of handicaps and cumulative advantages. The stability of the procedure is investigated by applying a tree to a subsequent sample.

The next example illustrates use of the procedure to locate interaction terms for inclusion in a multiple regression analysis. Interpretation problems from the inclusion of indices representing complex interactions as predictors are noted.

The final example (number of hours worked) provides another illustration of a two-stage analysis. Variables which were felt to be early in a possible causal chain (in the sense that they could influence
other predictors, but could not themselves be influenced by the other predictors) were put into the first stage. The results provide an interesting picture of constraints operating to reduce the number of hours worked, rather than of motivational factors.

When going through these examples, the reader should keep in mind that, unlike a multiple regression technique, this procedure allows predictors to substitute for each other in explaining variation in the dependent variable. Thus, when examining each split, the question, "What are the reasons why the split was made on this variable, rather than on one of the other predictors?" should be kept in mind.

Section 3.2 Average Hourly Earnings
A complete analysis is presented, illustrating various aspects of the revised computer program (AID Model 2), and several strategies which may be employed in the interpretation of the results. The objective was to replicate a previous analysis (24) of average hourly income. Some of the variables (e.g., place where head grew up) are multidimensional, since they were used previously in an additive model and interactions had been suspected between their components. A two-stage analysis was employed for the dual purpose of separating out exogenous factors from more current situational factors and providing a stringent test of the explanatory power of occupation. The latter was accomplished by putting it in with the second-stage predictors. A listing of the computer input, the complete output and supporting documents are included. (See Appendices K, L and M.)

An equivalent hourly earnings measure was computed for the heads of spending units for a national sample (the quotient of head's total wage income divided by hours worked $x$ 100). Where the head had no wage income, the value was assigned to this variable. These observations ( $N=451$ ) were omitted from the analysis.

A two-stage analysis strategy was adopted. A11 variables to be used in both stages were used as input to the program. These variables are identified and described in detail below. The following variables were used in stage one:

| Variable <br> Number | Name | Number of <br> Classes |
| :---: | :--- | :---: |
| 1 | Physical Condition of Head | 4 |
| 3 | Education of Head | 8 |
| 8 | Rank in School | 8 |
| 11 | Race | 2 |
| 12 | Age | 7 |
| 22 | Sex | 2 |
| 23 | Religion | 4 |
| 24 | N/Ach (need-achievement score) | 4 |
| 25 | Background (place where head grew up) | 6 |

Since a multi-stage probability sample with varying sampling fractions was used, the analysis employed weights attached to each observation to adjust for differences in sampling and response rates. At the end of stage one, residuals were computed. These residuals were used as the dependent variable (with the same sample weights) in stage two. The following variables were used in stage two:

| Variable <br> Number | Name | Number of <br> Classes |
| :---: | :--- | :---: |
| 2 | Geographic Mobility (number of states lived in) | 6 |
| $3 *$ | Education | 8 |
| 4 | Immigration (of head or father) | 3 |
| 5 | Occupation | 10 |
| 6 | Supervisory Responsibility on Job | 3 |
| 7 | Frequency of Unemployment | 8 |
| 9 | Religion x Church Attendance | 7 |
| 10 | Attitude toward work x N/Ach (achievement | 7 |
|  | motivation index) | 7 |
| $11 *$ | Race | 2 |
| 13 | Education difference between Head and Wife | 7 |
| 14 | Urban-Rural Migration | 5 |
| 15 | North-South Migration | 6 |
| 16 | Family Composition (sex, marital status, | 8 |
| 17 | number of children) | 8 |
| 18 | Plans to help parents and children | 4 |
| 19 | Interviewers' rating on ability to conmunicate | 4 |
| 20 | Size of place (city size) | 6 |

The variables used in stage one were suspected to be logically prior to those used in stage two. The starred items, Education and Race were used in both stages. They were included in the second stage on the hypothesis that they were likely to have both direct and indirect effects, and they were likely to interact with occupation in explaining variation in the residuals. The index of achievement motivation, and religion, were each reintroduced in combination with an allied variable.

The stage-one tree is presented in Chart 1. The total reduction in prediction error from these variables is .242 , which corresponds roughly to a multiple $\mathrm{R}^{2}$ of that size. Physical condition, rank in school, race and religion were not actually used by the program.

Stage one shows the powerful effects of education, age and sex. Achievement motivation appears important only for college graduates over 35 years of age. Rural-urban-north-south background appears important only for noncollege graduates.

## Structure of the Tree

After the initial division of the sample into three parts (groups 3, 4 and 5), the branching process follows a "trunk-twig" pattern. That is, successive branches isolate a subgroup, which is not split fur ther.

The reasons why these groups are not split further is of some theoretical importance. Either the number of observations is too small to warrant splitting the group, or the proportion of variation in it, compared to the variation in the total sample is too small, or no predictor in the analysis is capable of reducing the unexplained variation in that group the requisite amount.

If we consider groups 14, 21,23 and 25 , we find that the latter three are either too small to split, or do not have sufficient internal variation to warrant an attempted split. Group 14 cannot be split further, even though it has sufficient internal variation to warrant an attempt. No predictor "works." Age comes closest, but does not reduce the unexplained variation enough for the split actually to take place.

Two other groups, group 8 and group 19, did not have sufficient internal variation to warrant an attempted split, though they contained 95 and 73 persons respectively. For the other final groups, 7, 10, 16, 17, 12 and 18, no predictor "worked." If a group has a small variance, it has been explained. If it has a large one and no predictor works, then additional variables are needed in the analysis.

The tree illustrates a complex interaction between age, education, sex, N/Ach, and background. The trunk-twig structure indicates what
one might call the "alternative barrier" situation with respect to achieving high hourly income. If one is a college graduate, being under 35 years old is a "barrier," which cannot be surmounted by being characterized by any set of classes of the predictors used in the analysis, at least under the split criteria set up. If one passes this hurdle, then the absence of middle or high achievement motivation constitutes a barrier, etc. (see Table 1).

The same description applies to the noncollege-graduates who did not grow up on a farm or in the south. Being a woman (group 7), or being young (group 8), or failing to complete high-school (group 10), constitute alternative barriers (see Table 2).

Similarly, for noncollege-graduates who grew up on a farm or in the south, completing less than nine grades of school is a barrier, as is being a woman (see Table 3). Considering groups 7, 8, 10 and 16 , it is clear that there are different sets of barriers for men than there are for women, since group 7 (women) was not split further, though eligible (education was almost good enough to be used to split group 7).

This stage one tree illustrates the extent to which variables may substitute for one another in the analysis, depending on how they are correlated with the dependent variable. For instance, an examination of Table 1 indicates that the Urban-Rural-Farm-Nonfarm background of the Head was almost as good as Education in the split of group one into two and three. It was not used at that stage, but did not have its relationship to the dependent variable reduced enough by the split to prevent its being used in the split of group two into four and five. However, in group three, its relationship to the dependent variable has dropped considerably, and it was not used in further splits on this trunk. It appears important, from an analysis standpoint, to make a careful examination of those variables which were not used in the tree, but which, as it were, "almost made it."

Rank in School is another case in point. Examination of Table 2 indicates it was second-best in a number of branches, and would have been used if group 10 had been permitted to split by lowering the reducibility criterion.

## Education as a Substitute for Race

Another example of substitutability is the variable Race. There is plenty of evidence that being white or nonwhite affects one's wage rate. In this sample, the mean wage rate for whites is $\$ 2.38$, for nonwhites it is $\$ 1.60$. Moreover, in each of the final groups except one (see Table 4) there are white/nonwhite discrepancies between group means ranging from $\$ .11$ to $\$ 1.49$. Some of the $N^{\prime} s$ in these groups are too small to put much trust in, but the replicated discrepancies point overwhelmingly to an important race effect. Furthermore, the mean residuals for nonwhites are $-\$ .35$. If race exercised no effect, this would be closer to zero. Clearly there is a race effect. Why doesn't it show up in the tree?

We may reason as follows. Race may be considered to affect wage rates directly, and also indirectly, through its effects on other variables, which in turn affect wage rates. This combination of effects is undoubtedly quite complex and a detailed analysis is beyond the scope of this discussion. However, a discussion of race, education and wage rates will serve to illustrate an analysis strategy based on the algorithm.

We may hypothesize that race affects wage rates partly through its effect on education. Education is clearly a powerful predictor; but other things than race affect education. If this indirect effect is occurring, we should expect to find that nonwhites tend to have less education than whites. A stringent test of the hypothesis that this indirect effect is occurring would be to examine the relationship between race and education in each of our final groups. If nonwhites tend to have less education, the hypothesis of the existence of this indirect effect would be confirmed. An examination of the bivariate frequency distributions between race and education for each of the final groups tends to confirm this interpretation. In groups 14, 23, 25, $7,8,10,16,17$, and 12 , whites tended to have a higher proportion of individuals in the upper educational categories. For instance, in group 17, we find (percentages based on weighted data):

|  |  | Per cent <br> having only a <br> high-school <br> education <br> or less | Maving <br> additional <br> vocational <br> or college <br> training | Total |
| :--- | :---: | :---: | :---: | :---: |
| N | $45 \%$ | $55 \%$ | $100 \%$ |  |
| White | 297 | 62 | 38 | 100 |
| Nonwhite | 14 | 45 | 55 | 100 |

Group 21 had no nonwhites. In group 24 (college graduates) nonwhites had a slightly higher proportion of persons with advanced degrees. In this group, as in most of the others, however, the $N$ 's are relatively sma11. Groups 18 and 19 show a somewhat different pattern indicating that for rural and/or southern noncollege-graduate males, the pattern of relationships between race, education and sex is somewhat more complex. There is a larger proportion of high school drop-outs among nonwhites than among whites. Nonwhites who got education past high school tended to go to college rather than get other types of training. Perhaps there are a number of factors influencing the types of post high school education obtained by these males. The statistics for group 18 are as follows (percentages are based on weighted data).

|  | N |  | High School Graduate | High School plus Noncollege Training | $\begin{gathered} \text { Col1ege, } \\ \text { No } \\ \text { Degree } \end{gathered}$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| White | 410 | 45\% | 26\% | 13\% | 16\% | 100\% |
| Nonwhite | 58 | 60 | 10 | 6 | 24 | 100 |
| Total | 468 | 46 | 25 | 12 | 17 | 100 |

The fact that a very similar pattern is repeated in group 19 (females with similar backgrounds) lends credence to the complexity notion.

## Interrelationships between the Predictors

Additional hints as to the structure of interrelationships among the variables may be found in a manner similar to that used in constructing Table 4, by running frequency distributions on the predictors not used by the program. For instance 50 per cent of the American-born sons of immigrants are in groups 10 and 17 , approximately 25 per cent in each. The proportion of persons in group 10, high school drop-outs, scoring low, intermediate and high on the N/Ach predictor is, contrary to what might be expected, almost exactly the same as that for the total sample. Referring to Table 2, we see that Rank in School and Race were almost powerful enough to split group 10.

Groups 10, 12 and 18, and to a lesser extent, group 17, are similar in that they constitute relatively large numbers of respondents and are not splittable in terms of the algorithm and the split criteria. No single variable "works." And the analyst must consider the possible reasons why these groups could not be split. This suggests a possible revision of the program algorithm to consider the effects of each pair of predictors simultaneously for this type of group since there may exist negative, offsetting interactions. This might be done in either of two ways, which are similar, but not identical. One method would involve the treatment of a two-way analysis of variance table so that the methods outlined in the present algorithm are used on both the rows and columns simultaneously. An alternative would be to postpone the actual splitting process until the split rules which produce minimum within group variation in all possible "grandchildren" of the parent group under consideration, have been determined. This would constitute a "look-ahead" one step down each branch of the tree.

## Stage Two

Only two of the variables allowed as predictors in stage two were used, occupation and husband-wife educational differential. That occupation should pass this severe test of its effectiveness as a predictor is to be expected. The selection and use of the husband-wife educational differential is somewhat surprising. Group 5 was 36 per cent
female heads of spending units and 64 per cent males. All of the females from group 2 are located in group 5. Thus, the split reflects partly a male-female differentiation. In group 5, 26 per cent of the respondents are single males. Thus, 62 per cent of this group are single. The remainder are married male heads of spending units. As we might expect (see Table 5), family composition is almost as good a predictor as husband-wife educational differential, in the attempted split of group 2.

One way of interpreting this is to examine the nature of the two variables. Husband-wife educational differential may be considered to be tapping at least three sources of variation; sex, marital status among males, and husband-wife educational differentials among married males. Family composition taps only two of these sources, sex and marital status. But we note that in the program output detail for the split of group 2 into groups 4 and 5, that we do, apparently have an educational-differential effect, as is indicated below.
$\left.\begin{array}{lcc}\hline \hline & \text { Mean } & \text { N } \\ \hline \text { Education of Wife N.A. } & +.61 & 9 \\ \text { Wife has two or more levels } & & \\ \begin{array}{ll}\text { of education more than head }\end{array} & +.56 & 149 \\ \text { Wife has one more level than head } & +.39 & 244 \\ \text { Wife has same level } & +.27 & 496 \\ \text { Wife has one less level } & +.27 & 264 \\ \text { Wife has two or less levels } & +.04 & 251 \\ \text { No wife present (male and female) } & -.06 & 408\end{array}\right\}$

This variable apparently had no further effects in groups 6 and 7 (see Table 5), but after the farmers were separated out of group 3, it still showed some effects in groups 8 and 9.

## Summary

This example has been presented to illustrate the use of a twostage analysis to provide a stringent test of the effects of a variable which is known to be of considerable theoretical importance (occupation) and which has high correlations with other important variables, such as education.

The difference between two types of final groups, homogeneous or small, and unsplittable has been described.

The "trunk-twig" or alternative barrier tree structure as opposed to a more symmetric or "trunk-branch" structure, has been discussed.

Several examples of the substitutability of variables as a characteristic of the analysis algorithm have been presented and their implications for interpretation have been discussed. A strategy for investigating the extent to which a variable which has been used in a split is substituting for other variables is presented, together with its converse, a strategy for investigating why a variable which has considerable outside evidence as to its effects--does not get used.

It is recomended that all output options be exercised, including the punching of residuals as an aid to simplifying further analyses.

Some suggestions for further possible revisions in the analysis algorithm are made.

## CHART 1

AVERAGE HOURLY EARNINGS*


RESIDUALS - AVERAGE HOURLY EARNINGS*


Reduction in error
as a proportion of residual $T S S=.114$
as a proportion of original $T S S=.087$
*For heads of spending units who had
income in 1959
SOURCE: Survey Research Center
Study 678
719 MTR 51

Total reduction in error
from two-stage analysis as a proportion of original $\mathrm{TSS}=.329$

Table 1
WAGE RATE ANALYSIS STAGE 1 COLLEGE GRADUATES ONLY


Proportion of variation in that group explainable for each predictor (BSS/TSS) ${ }_{i}$
$\rightarrow$ Split made on this variable.
xXX $=$ Next best BSS/TSS.
${ }^{x}=$ Final group.
$\Upsilon=$ Split attempted but not made.
$N A=$ Split not attempted.
Source: ISR Study 678, Deck 35 719, MTR 51

Table 2
WAGE RATE ANALYSIS STAGE 1
NONCOLLEGE GRADUATES WHO DID NOT GROW UP ON A FARM OR IN THE SOUTH

|  | Group Number |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 4 | 7* | 6 | 8* | 9 | 10* | 11 | 16* | 17* |
| Physical condition | . 015 | . 014 | . 003 | . 028 | . 002 |  | . 003 | . 005 | . 008 | . 007 | . 008 |
| Education | . 083 | . 049 | . 023 | . 092 | (029 |  | . 040 | . 009 | . 003 | . 027 | . 002 |
| School rank | . 041 | . 022 | . 016 | 076 | . 023 |  | 025 | ¢ .019 | 013 | . 032 | (012) |
| 'Race | . 027 | . 027 | . 011 | . 017 | . 009 |  | . 009 | 017 | . 004 | . 009 | . 000 |
| Age | . 020 | . 016 | 038 | . 017 | . 032 |  | . 009 | . 013 | . 033 | Const. | .013 |
| Sex | . 047 | 053 | . 086 | $\begin{aligned} & \text { Const. } \\ & \text { F } \end{aligned}$ | Const. M |  | Const. <br> M | Const. <br> M | Const. <br> M | $\begin{gathered} \text { Const. } \\ M \end{gathered}$ | $\begin{aligned} & \text { Const. } \\ & \mathrm{M} \end{aligned}$ |
| Religion | . 030 | . 027 | . 012 | . 070 | . 014 |  | . 012 | . 014 | (013) | . 023 | . 005 |
| N/Ach | . 017 | . 013 | . 007 | . 036 | . 010 |  | . 010 | . 008 | . 008 | . 047 | . 006 |
| Background | $067$ | . 060 | ${ }_{.003}$ | . 006 | . 004 |  | . 005 | . 004 | . 000 | . 016 | . 003 |
| N | 2546 | 2262 | 1244 | 207 | 1037 | 95 | 942 | 477 | 465 | 154 | 311 |
| $\mathrm{TSS}_{\mathrm{i}} / \mathrm{TSS}_{T}$ | 1.0 | . 738 | . 415 | . 022 | . 358 | . 013 | . 334 | . 146 | . 174 | . 034 | . 134 |
| MEAN | 2.31 | 2.16 | 2.43 | 1.56 | 2.60 | 1.84 | 2.67 | 2.41 | 2.94 | 2.58 | 3.11 |

Proportion of variation in that group explainable for each predictor (BSS/TSS) ${ }_{i}$
$\rightarrow$ Split made on this variable.
(xxx $=$ Next best BSS/TSS.
${ }^{*}=$ Final group.
N = Split attempted but not made.
$N A=$ Split not attempted.
Source: ISR Study 678, Deck 35
719, MTR 51

Table 3
WAGE RATE ANALYSIS STAGE 1
NONCOLLEGE-GRADUATES WHO GREW UP ON A FARM OR IN THE SOUTH

|  | Group Number |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 5 | 12* | 13 | 18* | 19* |
| Physical condition | . 015 | . 014 | . 033 | . 026 | . 017 | . 015 |  |
| Education | . 083 | . 049 | . 059 | . 009 | . 004 | . 006 |  |
| School rank | . 041 | . 022 | . 026 | (028) | . 002 | . 003 | NA |
| Race | . 027 | . 027 | . 027 | . 014 | . 020 | . 011 |  |
| Age | . 020 | . 016 | . 016 | .017 | 019 | . 020 |  |
| Sex | . 047 | .053 | 034 | $.038$ | . 033 | Constant |  |
| Religion | . 030 | . 027 | . 011 | . 005 | . 007 | . 011 |  |
| $\mathrm{N} / \mathrm{Ach}$ | . 017 | . 013 | . 017 | . 011 | . 015 | 017 |  |
| Background | . 067 | . 060 | . 018 | . 004 | . 011 | . 020 |  |
| N | 2546 | 2262 | 1018 | 477 | 541 | 468 | 73 |
| $\mathrm{TSS}_{\mathbf{i}} / \mathrm{TSS}_{\text {T }}$ | 1.0 | . 738 | . 278 | . 094 | . 167 | . 153 | . 009 |
| MEAN | 2.31 | 2.16 | 1.77 | 1.41 | 2.03 | 2.12 | 1.41 |

Proportion of variation in group explainable for each predictor (BSS/TSS $_{i}$.
$\longrightarrow=$ Split made on this variable.
$x \times x)=$ Next best BSS/TSS.

* $=$ Final group.
i = Split attempted but not made.
$\mathrm{NA}=$ Split not attempted.
Source: ISR Study 678, Deck 35
719, MTR 51

Table 4
WAGE RATE ANALYSIS STAGE 1
MEAN INCOME BY RACE WITHIN GROUP

| Group | N | White |  | Nonwhite |  | Discrepancy |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mean | N | Mean | N |  |
| 14 | 97 | 2.87 | 93 | 3.64 | 4 | -. 77 |
| 21 | 25 | 2.67 | 25 | -- | 0 | -- |
| 23 | 20 | 2.86 | 19 | 2.65 | 1 | +. 11 |
| 24 | 133 | 4.29 | 125 | 2.80 | 8 | +1.49 |
| 25 | 11 | 2.69 | 10 | 1.43 | 1 | +1. 26 |
| 7 | 207 | 1.60 | 180 | 1.28 | 27 | +. 32 |
| 8 | 95 | 1.86 | 87 | 1.58 | 8 | +. 28 |
| 10 | 477 | 2.45 | 439 | 1.77 | 38 | +. 68 |
| 16 | 154 | 2.62 | 135 | 2.30 | 19 | +. 32 |
| 17 | 311 | 3.12 | 297 | 3.00 | 14 | +. 12 |
| 12 | 477 | 1.49 | 328 | 1.17 | 149 | +. 32 |
| 18 | 468 | 2.16 | 410 | 1.71 | 58 | +. 45 |
| 19 | 73 | 1.54 | 51 | . 92 | 22 | +. 62 |
| Total | 2548 | 2.38 | 2199 | 1.60 | 349 | +. 78 |

Source: ISR Study 678, Deck 35
719, MTR 51

Table 5
WAGE RATE ANALYSIS STAGE 2
RESIDUALS

|  | Group Number |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 5* | 4 | 6* | 7* | 3 | 8* | 9* |
| Geogr. Mobil. | . 007 | . 007 | . 005 | . 008 | . 011 | . 004 | . 005 | . 005 | . 006 |
| Education | . 002 | . 001 | . 008 | . 006 | . 006 | . 004 | . 007 | . 017 | . 009 |
| Immigr. | . 000 | . 000 | . 001 | . 000 | . 003 | 008 | . 000 | 000 | . 006 |
| Occup. | . 084 | . 010 | . 016 | . 010 | . 0001 | . 000 | . 064 | .001 | -- |
| Supv. Resp. | . 017 | . 008 | . 007 | . 007 | . 007 | . 002 | . 041 | . 003 | . 012 |
| Freq. Unempl. | . 023 - | . 007 | . 0212 | . 002 | . 003 | . 007 | . 023 | . 006 | . 004 |
| Rel. x Att. | . 008 | . 006 | . 013 | . 009 | . $012 \square$ | . 016 | . 007 | . 012 | . 016 |
| Work x N/Ach | . 006 | . 002 | . 009 | . 002 | . 001 | . 003 | . 008 | . 012 | . 010 |
| Race | . 009 | . 003 | . 005 | . 002 | . 000 | . 005 | . 000 | . 003 | . 000 |
| H-W Educ. | . 012 | . 019 | S 002 | $\xrightarrow{.} 005$ | . 008 | . 005 | . 005 | . 022 | . $029<$ |
| Urb-Rur Mig. | . 013 | . 008 | . 011 | . 007 | . 005 | . 0262 | . $045 \times$ | . 0372 | . 010 |
| $\mathrm{N}-\mathrm{S}$ Mig. | . 006 | . 005 | . 004 | . 007 | . 010 | . $017 \times$ | . 025 | . $025 \square$ | . 0512 |
| Family Comp. | . 016 | . $017 \times$ | . $020<$ | . 004 | . 007 | . 003 | . 008 | . 014 | . 012 |
| He1p Par. \& Child | . 011 | . 006 | . 009 | . 001 | . 001 | . 004 | . 006 | . 002 | . 029 |
| Comm. Abil. | . 008 | . 002 | . 002 | . 005 | . 006 | . 003 | . 001 | . 003 | . 010 |
| Size of Place | . 029 < | . 007 | . 019 | . 007 | . 0132 | . 003 | . 042 | . 013 | . 006 |
| H-Fa. Educ. D. | . 002 | . 001 | . 011 | . 000 | . 004 | . 010 | . 002 | . 006 | . 009 |
| N | 2546 | 1821 | 659 | 1162 | 661 | 501 | 725 | 528 | 197 |
| TSS ${ }_{\text {i }} / \mathrm{TSS}_{\text {t }}$ | 1.0 | . 748 | . 225 | . 509 | . 370 | . 134 | . 168 | . 098 | . 060 |
| MEAN $t$ | . 783 | . 205 | -. 021 | . 336 | . 452 | . 185 | -. 653 | -. 495 | -1.110 |

Proportion of variation in each group explainable for each predictor (BSS/TSS) ${ }_{i}$
$\longrightarrow=$ Split made on this variable.
$\square=$ Next-best BSS.

* $=$ Final group.
$\mathcal{Z}=$ Split attempted but not made.
- = Variable is constant in this group.

Source: ISR Study 678, Deck 35
719, MTR 51

Section 3.3 A Dichotomous Dependent Variable--Home Ownership
Home ownership in early 1959 was analyzed using data from the 1959 Survey of Consumer Finances (25) in which 2980 nonfarm spending units were interviewed. They were weighted to account for varying sampling and response rates. The explanatory classifications allowed (all free to be rearranged) were:

| Characteristic | Number of subclasses, <br> including missing <br> information |
| :--- | :---: |
| Age of head of unit | 7 |
| Number of people in the unit | 10 |
| Income | 9 |
| Education of head of unit | 7 |
| Race | 4 |
| Number of "major" earners (\$600 or more) | 6. |
| Whether income last year was unusual <br> (a combination of reported income change, <br> unemployment in 1958, and whether head <br> was in the labor force) | 8 |

The eight final groups in the tree (see Chart 3) accounted for some 23 per cent of the total sum of squares, making use of only three of the seven factors: age, income, and number of people in the unit. A previously-run multiple regression using the same predictors found age, income, number of persons, race, and "whether last year's income was unusual" all significant, and explained the same fraction of the total sum of squares. According to either analysis, the proportion of home owners increases with age, with additional influences from higher income and larger families. What the tree adds is the impression that it takes a wife and children to push the young to home ownership, and then only if their income is adequate, whereas among the older people marriage is enough, with the single people becoming home owners mostly after they are 55 years old.

It is certainly more economical to explain home ownership with eight combinations of three characteristics, rather than the 45 subclasses of seven predictors used in the regression. More important, we are kept from assuming that there is a single uniform effect of family size on home ownership, or of age on home ownership. Interestingly enough, however, the best income division to discriminate older couples as to their home ownership was the same as the best division for younger families (most of which have children).

It should be noted that even though age was used in an early split, it was still eligible to be used again, and was used in a later split. The program does not discard a variable after using it once.

## CHART 3

HORE OWNERSHIP IN EARLY 1959
BY CHARACTERISTICS OF SPENDING UNITS


[^0]An example (26) of a relatively simple scaled dependent variable resulted from asking 2384 people who were in the labor force in August 1962 or November 1962 the questions:

Do you think there is any chance you will move away from (town or place where now living) in the next year?
If some chance: Would you say you definitely will move, you probably will, or are uncertain?

Those who said they definitely or probably would move were coded " 2 ," those who were uncertain were coded " $1, "$ and those who indicated little chance of moving were coded "0." The assumption is made explicit that these points are deemed to represent approximately equal intervals on an underlying continuum "probability of moving."

The prior multiple regression analysis was done separately for four subgroups on the assumption that there might be interaction effects, i.e., that other factors might operate differently on each of them. The four were:

## Mean

Score
. 22 People under 35 years of age living in a redevelopment area
. 28 People under 35 years of age living elsewhere
. 10 People 35 or older living in a development area
. 11 People 35 or older living elsewhere

There appeared a tendency for one variable (having relatives living nearby) to affect mostly the young. Another (whether moved in last five years) affected mostly those not in redevelopment areas. Two variables (whether unemployed last year, whether owns home) tended to affect only those 35 and over and not in a redevelopment area. One (whether a college graduate) affected only those under 35 not in a redevelopment area and one (being very young, 18-24 years old) affected only those under 35 and in a redevelopment area.

Chart 4 using the same explanatory factors, gives quite a different impression. Neither of the two factors assumed to be crucial in
the four regression subgroups appear in the tree. The first split makes use of past mobility (which was significant in only two of the regressions). The variables used were:

| Variables |
| :--- |
| Age of head of the spending unit |
| Education of head |
| Whether a redevelopment area (a county or pair of counties |
| designated by the Area Redevelopment Administration as |
| having sufficiently low income or sufficiently high |
| unemployment to qualify for assistance) |

Having children in school, which appeared significant in none of the regressions, makes an important split among those who have moved in the past five years. Other splits use car ownership, significant nowhere in the regressions, and education which was significant only in one.

Two problems are apparent with this tree. First, the combinations of education are difficult to interpret. Second, and more important, there is some circularity in using past mobility to explain expected mobility. For predictive purposes this may be all right, but it does not "explain" mobility.

A second analysis was made, omitting only "whether moved in the past five years," and is presented in Chart 5. Instead of the full
tree with the . 005 reducibility criterion, this tree has been truncated (some very small final groups were combined into their parent). The results have an intuitive appeal to them, and provide a vivid impression of alternative inhibitors of moving: age, relatives nearby, children in school, or owning a home.

In such a situation, the particular sequence of splits may well be unstable, since once one factor is used, the other can only influence the nonhampered group. Subsequent analysis might well be done developing a new variable: "Any one of the following inhibiting factors is present," and would involve an analysis of the correlations between the predictors.

CIIART 4
PLANS TO MOVE
0 . No chance of moving

1. Uncertain or depends
2. Definitely or probably will move
3. Definitely or probably will move

## CHART 5

PLANS TO MOVE
0 . No chance of moving

1. Uncertain or depends
2. Definitely or probably will move


Section 3.5 A Skewed Distribution-Nonfamily Contributions
Dollar contributions during 1959 reported by families as made to charity, church, and relatives not living in the household, had been analyzed by using an additive dummy-variable regression technique (27).

The prior analysis used some variables representing interactions between the original classifications--combinations of religious preference and church attendance, and combinations of race, age, education and farmer status entitled "earning potential."

The badly skewed nature of the dependent variable had been ignored in the original analysis, but showed up immediately in the AID results. Sixteen of the twenty-two final groups contained ten or fewer observations. Eliminating 33 cases of the original 2800 where contributions of $\$ 3,000$ or more were reported, reduced the standard deviation of the dependent variable from $\$ 725$ (mean was $\$ 315$ ) to $\$ 419$ (with mean of $\$ 254$ ).

Table 6 gives the classifications used, which were purposely kept the same even in a second AID run which excluded the 33 extreme cases. Neither of the trees is given here because they are difficult to read. In addition to the problem of small groups split off, which remained even after eliminating the most extreme cases, the introduction of complex classifications such as "earning potential" into the AID analysis lead to combinations of combinations which were extremely difficult to describe and interpret. A revised program allowed us to constrain such factors as "number of children" against reordering of the scale.

Consequently, a third AID run was made, using the components of the complex classes separately: religion, church attendance, race, age, education, labor force status. The results are given in Table 7 and in Chart 6. There is a clear preponderance of income as an explanatory factor, but also a clear tendency for those over 45 years old to contribute more to others.

The problem of skewness still remains, as can be seen from the two remaining cases where a group of two or three is split off, reducing the error sum of squares by more than 1 per cent in each case.

An examination of the extremely large contributors revealed that they tended to have quite high incomes and either dependent parents or
children living away from home (in college, just married) to whom they were making gifts. For very high income people such gifts are an important method of avoiding estate and inheritance taxes. The persistence of small groups would indicate that some transformation of the dependent variable into logs or percentages of income might be necessary. There are disadvantages to any of these transformations, however, when it comes to interpreting the results.

It is not the purpose here to provide a thorough analysis and interpretation of the results of each of these exploratory analyses. Two general questions are always to be asked:
I. At any stage, are there competing factors correlated with the one actually used in the split, and subsequently made unimportant?

In this case, an analysis of the between-sums-of-squares for the best split on each predictor at each stage indicates that whenever a second factor was almost as good as the one used, it tended to come into its own and be used later on in at least one of the branches. This, however, must necessarily be a property of the particular set of variables used in the analysis and depends on the orthogonality of the predictors.
2. Do the results suggest hypotheses which, for final testing, would require new information?

The importance of age in the analysis, with those over 45 persistently contributing more, raises questions whether this is the result of more assets, or more children, relatives and organizations making claims on older people, or whether it reflects a historical process of the passing away of private philanthropy--the younger generation being more willing to leave it to the government. No data are available on the existence of relatives who need aid. "The attitudes toward government responsibility for the aged, and toward level of unemployment compensation benefits are not strikingly different between the young and old (24).

One of these attitudes was used directly in the analysis and comes in only at the end and with older people.

The output of the AID program gives the subgroup means in detail at each split so that one can observe whether anything was lost by
maintaining the order of the age groups. Nothing was, since only one small age group would have switched to the other side. It is also possible to look at the competing factors at each stage to see which factors nearly succeeded. Near the ends of the trees there were some cases where geographic background and current marital status nearly "made it," but in both cases the $N$ 's were quite small.

The importance of church attendance is not surprising, though there are problems whether it is cause or effect, or a joint result of some more basic factor.


Table 6
PROPORTION OF VARIATION IN NONFAMILY CONTRIBUTIONS EXPLAINED BY FAMILY CHARACTERISTICS

| Characteristics of families | Type | $\begin{gathered} \text { Al1 } \\ \text { families } \end{gathered}$ | Families with contributions of \$0-2999 only | Squared Beta coefficients from additive regression analysis* |
| :---: | :---: | :---: | :---: | :---: |
| Gross disposable income | F | . 190 | . 187 | . 173 |
| Earning potential of heads | F | . 105 | . 028 | . 016 |
| Religious preference and church attendance of heads | F | . 053 | . 022 | . 012 |
| Number of living children of heads | F | . 031 | . 016 | . 004 |
| Political preference | F | . 063 | . 009 | . 004 |
| Age of heads at birth of eldest living child | F | . 016 | . 000 | . 002 |
| Number of siblings of heads | F | . 039 | . 006 | . 002 |
| North-South-Urban-Rural background of heads | F, | . 006 | . 016 | . 001 |
| Attitude of heads toward who should have primary responsibility for aged; government or relatives | F | . 000 | . 006 | . 001 |
| Sex of heads | F | . 000 | . 000 | . 001 |
| Family provides housing for nonnuclear family members in household | F | . 031 | . 000 | . 000 |
| Total proportion of variation explained |  | . 534 | . 290 | . 22 ** |
| Mean contributions |  | \$315 | \$254 |  |
| Standard deviation of contributions |  | \$725 | \$419 |  |
| Number of observations |  | 2800 | 2767 |  |

[^1]Table 7
PROPORTION OF VARIATION IN NONFARMILY CONTRIBUTIONS
EXPLAINED BY FAMILY CHARACTERISTICS
(Contribution of $\$ 0-2999$ only)

| Characteristics of families | Type | Number of classes | Proportion of variation explained |
| :---: | :---: | :---: | :---: |
| Family gross disposable income | M | 10 | . 179 |
| Marital status of heads | F | 6 | . 000 |
| Labor force status of heads | F | 7 | . 000 |
| Age of heads | M | 7 | . 030 |
| Sex of heads | F | 2 | . 000 |
| Race of heads | F | 4 | . 000 |
| Education of heads | M | 8 | . 007 |
| Number of siblings of heads | M | 5 | . 000 |
| Number of living children of heads | M | 5 | . 000 |
| Attitude of heads toward government or relatives having responsibility for aged | F | 7 | . 018 |
| Religion of heads | F | 10 | . 032 |
| Church attendance of heads | M | 7 | . 027 |
| North-South-Urban-Rural background of heads | F | 6 | . 011 |
| Family provides housing for nonnuclear-family members in household | F | 2 | . 000 |
| Total proportion of variation explained |  | . 304 |  |
| Mean contributions |  | \$254 |  |
| Standard deviation of contributions |  | \$419 |  |
| Number of observations |  | 2767 |  |

Data from Friedman, Whelpton and Campbell (28) were used in an analysis of expected family size. The input consisted of responses from all wives married ten or more years, whose fecundity was not classified as indeterminate. Three analyses were run, as illustrated in Charts 7, 8 and 9.

The first analysis (see Chart 7) included twelve predictors (see Table 8). All were left free in mode, that is, the class orderings were not constrained. The analysis explained thirty-seven per cent of the variation in the dependent variable, with number of years worked by wife, husband's education, fecundity status, husband's occupational status, wife's education, and an interaction of religious preference and attendance accounting for over thirty per cent of the variation. The results are generally in conformity with those reported by Friedman, Whelpton and Campbell. However, this tree serves to illustrate several properties of the AID algorithm. This analysis contained variables with two classes (fecundity, wives' age at marriage) to ten (wives' work experience, husband's occupational status, and education of both husband and wife). The tree indicates that wives who have worked zero through three years have a mean of 3.5 on the dependent variable, and those who have worked four or more years have a mean of 2.2. But the interpretation of the extremely powerful effect of this variable is difficult. It taps variation associated with the work-enabling situation of sterility and/or children in school. It may well be the result of a decision to work rather than care for more children. This decision is a complex function of attitudes toward family size limitations, economic aspirations, attitudes toward the appropriate role of an adult woman, job opportunities, etc. Thus, it may be interpreted as an effect of family size, rather than a link in a causal chain explaining family size. Family size may be an enabling condition for working.

These issues arise because of the question which should be asked at each split. "Why should this variable be more highly correlated with the dependent variable than any other one in the analysis for this particular group?" The answer may be that this variable is very highly correlated with one or more other variables which have not been
measured directly, and which are very close to the dependent variable in a causal chain, either as a cause or as an effect. Another answer is that the more classifications (in this case, ten) encompassed by a variable, the more likely it is for the algorithm to find a permutation of the class means that will produce a high between-groups sum of squares. However, constraining the order of the classifications would not, in this case, have caused another variable to be used at this stage.

The same type of problem may be seen later on in this tree in the behavior of the variables wives income, husbands occupational status, wives education and husbands education. Husbands occupational status is a derived measure based on occupation, salary, and education, for which a score between 0 and 99 is computed. The measure coded for use as a predictor consists of the ten deciles of that score distribution. On this basis, the splits in the tree do not make sense. When a relatively small group is partitioned on the basis of an unconstrained predictor with a large number of categories, the sampling variation of the class means will be large because of the small number of observations in each class. The probability of a fortuitous split is relatively high.

We are led to a conservative rule of thumb. Predictors which have a rank ordering to their classifications should be constrained to that ordering during the partition process, and unordered predictors should not have more than five or six classes. The exception to the rule of constraining rank ordered predictors is the case where the possibility of a U-shaped or inverted $U$ relationship between that predictor and the dependent variable is suspected, in which case adjoining classes should be combined to form a maximum of five and the variable left unconstrained.

Charts 8 and 9 are identical runs except that all predictors are unconstrained in the first run (8), and both education variables, husband and wife have a constrained status in the second run. Also, in Chart 9 no group where $N_{i}<25$ was permitted to split. Both runs used six predictors, a subset of the predictors listed in Table 8. They were husband's education, wife's education, size of city lived in,
attitudes toward family limitation, fecundity status and the interaction variable religion and attendance.

In Chart 8 , the tree produced an $R^{2}$ of .259 as compared with .216 in Chart 9. Here, we have a clear effect of the constraints on the ordering on the ranked variables having a large number of categories. In Chart 8, one suspects that the later splits on education tend to be susceptible to influence by sampling variation. The constraints are not present. There are more final groups in this tree. Variation is being attributed to education which probably does not belong there. The fact that several splits appeared in which a very small group was separated from a large one leads one to suspect a skewed or very spread out rectangular distribution. These extreme observations should undoubtedly be subjected to a careful deviant case analysis to see if they have something in common that is not used as a predictor in the tree.

Other somewhat unexpected findings appear, and are associated with the interaction variable religion $x$ attendance. The expected relation between Catholicism, church attendance and expected family size is not found. Regular attenders who are Catholic show up as having fewer expected number of children than those who only report attending often. These may represent measurement errors, sampling errors, or a genuine finding.

There appears to be evidence in the tree presented in Chart 8 that the variable place of residence is somewhat differently related to expected family size in the three subgroups in which it was used as a criterion for splitting. Table 9 illustrates the differential behavior of the variable. In the total sample of wives married ten or more years, the clearest difference is between the rural farm wives vs. the remainder. This is also characteristic of group A, the sterile wives, and group $B$, the fecund wives with 9 or more grades of education who do not disapprove of family limitation. Group B is most like the total sample. The effect of sterility is clearly shown by an examination of the lowered means in group A, compared to the total group. Its effect is more pronounced with increasing urbanization. But in both group A and group B, the maximum binary split was the rural farm vs. all others.

A somewhat different pattern appears in group C, fecund wives with only one through eight grades of education. Here it is the twelve largest central city and suburban people who are quite different from the remainder. The somewhat surprising change in the rank-ordering of the means in this group, between the small towns and cities over 50,000 is consistant with results found by Friedman, et al., and may be explained by the fact that the place-code for metropolitan areas other than the twelve largest include the entire county in which the central city of over 50,000 is located, and probably contain uneducated persons who should more properly be classified as rural farm and rural nonfarm.

The implication of this finding for the further use of the algorithm is that in the initial stages of analysis, it may be desirable to leave all predictors unconstrained, and to use the program as a device for locating conceptual problems. It is quite likely that classes such as the $50,000+$ code for place of residence which; when used as an index of urban-rural residence do not conceptualize all of the population properly. In this case, it is probably true that those living outside the city of $50,000+$, but inside the county in which it is contained, are really living in a rural-farm or rural-nonfarm community situation. It is also quite likely that there is a fairly heavy concentration of loweducation people in these areas outside these small central cities. Thus, it is implied that the urban-rural variable, as coded, tends to place better educated persons more accurately along the rural-urban dimension than persons with low education.

One possible use for the procedure is to scan the data for variables which do not "behave" as expected. When unexpected findings appear, one possible interpretation involves the relation between the indicator, or variable used as a predictor and the underlying concept which it operationalizes. There may be some classes of the sample for which the variable does not correspond to the concept. One must, of course, decide whether the split represents covariation, conceptualization, coding errors, sampling variability or a genuine finding.

The purpose of this discussion has been to focus on the need for a careful examination of the relationship between the underlying concept and the indicator (predictor) as it behaves in the analysis.


CHART 8

EXPECTED NUMBER OF CHILDREN (IHCLUDING THOSE ALREADY BORN)


## Table 8 <br> RELATIVE POWER OF VARIABLES PREDICTING EXPECTED NUMBER OF CHILDREN

| Predictor | AID reduction in TSS (I)/TSS (T) | Number of classes |
| :---: | :---: | :---: |
| Number of years wives have worked | . 097 | 10 |
| Husbands education | . 072 | 10 |
| Wives fecundity status | . 044 | 2 |
| Husbands occupational status | . 041 | 10 |
| Education of wives | . 030 | 10 |
| Religion x attendance of wives | . 027 | 5 |
| Attitude of wives toward family limitation | . 018 | 5 |
| Farm background of husbands and wives | . 015 | 4 |
| Wives income | . 012 | 8 |
| Age of wives at marriage | . 008 | 2 |
| Present place of residence (urban-rural) | . 007 | 5 |
| Discrepancy in income between husbands and wives | . 000 | 3 |
| $\mathrm{R}^{2}$ | . 371 |  |
| Mean 3.09 |  |  |
| $\sigma$ 仿 1.92 |  |  |
| N |  |  |
| Source: ISR Project 719, MTR 26 |  |  |

Tab1e 9
MEAN EXPECTED FAMILY SIZE FOR THREE GROUPS, BY SIZE OF PLACE OF RESIDENCE

|  | Total |  |  | A |  |  | B |  |  | C |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | $\overline{\mathrm{Y}}$ | $\sigma$ | N | $\overline{\mathrm{Y}}$ | $\sigma$ | N | $\overline{\mathrm{Y}}$ | $\sigma$ | N | $\bar{Y}$ | $\sigma$ |
| Rural farm | 135 | 4.0 | 2.2 | 40 | 3.6 | 2.3] | 49 | 3.8 | $1.9]$ | 36 | 4.9 | 2.4 |
| Rural nonfarm | 189 | 3.3 | 2.0 | 54 |  | $-2.0$ |  | 3.2 | 1.6 | 37 | 4.4 | $2.2-$ |
| Places of 2500-49,999 | 180 | 2.9 | 1.7 | 55 |  | -1.6- |  | 3.0 | $1.4-$ | 20 | 3.7 | -2.1 |
| Cities 50,000+ and suburban rings which are not 12 largest cities | 396 |  | 1.8 | 88 |  | -1.8- |  | 2.7 | 1.4 | 47 |  | -2.2 |
| 12 largest central cities and suburban rings | 238 |  | 1.7 | 62 | 1.9 | 1.3 |  | 3.0 | 1.5 | 22 | 2.8 |  |
| Total--all wives married 10 years or more | 1138 | 3.1 | 1.9 | 299 | 2.3 | 1.9 | 566 | 3.0 | 1.5 | 162 | 4.1 | 2.3 |
| Group A Sterile wives married 10 or more years |  |  |  |  |  |  |  |  |  |  |  |  |
| Group B Fecund wives married 10 or more years with 9 or more grades of education who do not disapprove of family limitation |  |  |  |  |  |  |  |  |  |  |  |  |
| Group C Fecund wives married 10 or more years with 1 through 8 years of education <br> Groups placed together in the partition process |  |  |  |  |  |  |  |  |  |  |  |  |
| Source: ISR Project 719, |  |  |  |  |  |  |  |  |  |  |  |  |

Section 3.7 Average Number of Grades of School Completed by Children in the Spending Unit

A major vehicle for transmission of economic status from one generation to the next is formal education. A previous multivariate analysis (24) using the dummy-variable regression model, employed the explanatory factors listed in Table 10.

Table 10 gives the beta coefficients from a multiple classification analysis (22), squared for comparability of dimension, and the proportionate reduction in error sum of squares attributable to the same predictors when used in the AID analysis. It is interesting that every one of the predictors is used to make at least one split in Chart 10. This suggests that there really are many forces at work which are not so highly correlated with one another that the division of the sample on one makes the other unnecessary.

Again, however, there are problems when variables which themselves represent interactions are used, since the resulting splits involve combinations of combinations, frequently difficult to interpret. There are also some relatively small groups split off. However, most of the splits go in the expected directions.

In the right center of the chart is an interesting sequence in which first, those with a high index of achievement motivation are split off, and among the rest, those who go to church frequently (or are nonChristian). Are transmitted achievement motivation and a religiously oriented sense of responsibility alternative forces inducing people to provide more education for their children?

In a number of places one may wonder whether the variable used is really a proxy for one of the others, i.e., "grew up in the deep South and stayed there," meaning "mostly nonwhites." The program as now set up, provides a distribution on each predictor at each split so that one can tell to what extent a competing variable came close to being used.

There is a group where the father had some college training and was a professional, manager, self-employed, or government employee where children of fathers 55 and older had clearly more education than those under this age, and where an examination of the group before splitting indicated a continuous trend across five age groups. One
implication is that while in earlier generations the children of college educated fathers were almost certain to go to college, the strength of this effect has been getting smaller. (If colleges rely more on merit and grades and admit fewer of the "gentlemen" school, this finding might be real.)

Table 11 describes the final groups resulting from the AID analysis listed in decreasing order of the mean education of children in that group. The distribution of educational levels for spending units with living children who have completed their education is presented in Table 12.


Table 10

## AID AND MULTIPLE CLASSIFICATION ANALYSIS OF AVERAGE COMPLETED EDUCATION OF CHILDREN



Table 11
COMPLETED EDUCATION OF GHILDREN
FINAL (TRUNCATED) GROUPS IN RANK ORDER
BY THEIR AVERAGES

| Group number | Number of cases | Average years of education | Characteristics of parents |
| :---: | :---: | :---: | :---: |
| (30) | 63 | 15.2 | Father had some college and is a professional, or manager, or government employee or is self-employed and is aged 55 or older. |
| (23) | 35 | 13.7 | Father is professional, manager, or government employee who finished high school, but had no college. |
| (31) | 14 | 13.4 | Father had some college, is a professional, or manager, or government employee or is self-employed and is 35-54 years of age. |
| (20) | 69 | 13.4 | Father finished high school or has additional education and is blue collar. worker or clerical and is 55-74 years of age. |
| (6) | 59 | 13.0 | Father did not finish high school, did not grow up and remain in the South, mother had two or more levels of education than the father. |
| (21) | 77 | 12.1 | Father finished high school or has additional education, is blue collar worker or clerical and is aged 25-54 or over 74 years of age. |
| (16) | 48 | 12.1 | Father did not finish high school, did not grow up and remain in the South, was unskilled worker or farmer, did not have two or less levels of education than mother, had a highest income that was not in the lowest category, and scored high on achievement motivation. |

Table 11--(CONTINUED)

| Group number | Number of cases | Average years of education | Characteristics of parents |
| :---: | :---: | :---: | :---: |
| (8) | 145 | 12.0 | Father did not finish high school, did not grow up and remain in the South, did not have two or less levels of education than mother, was a white collar, skilled worker, or government employee. |
| (18) | 108 | 11.3 | Father did not finish high school, did not grow up. and remain in the South, did not have two or less levels of education than mother, had not always had low income, was low on achievement motivation, was a Christian and attended church regularly or was non-Christian. |
| (14) | 90 | 10.5 | Father did not finish high school, grew up in the South and stayed there, was not a laborer. |
| (19) | 107 | 10.4 | Father did not finish high school, did not grow up and remain in the South, did not have two or less levels of education than mother, was unskilled worker or farmer, had not always had low income, was low on achievement motivation, was a Christian who attended church infrequently. |
| (26) | 26 | 10.2 | Father did not finish high school, grew up in the South and stayed there, was a laborer, had highest income over $\$ 3000$. |
| (13) | 48 | 9.7 | Father did not finish high school, did not grow up and remain in the South, did not have two or less levels of education than mother, was unskilled worker or farmer, had a very low highest previous income. |
| (27) | 50 | 7.4 | Father did not finish high school, grew up in the South and stayed there, was a laborer, had never earned more than $\$ 3000$. |

Table 12

COMPLETED EDUCATION OF CHILDREN
OF SPENDING UNITS EXISTING IN EARLY 1960** (FOR THOSE WHO HAVE LIVING CHILDREN)


Section $3.8 \quad$ A Somewhat Skewed Variable--
An alternative to separate analyses of components of income, such as labor force employment of each member, hours of work, and hourly earnings, is to analyze the resulting combination of incomes, even though the causes may work through one or more of the components. It is important sometimes to see just what are the most important forces affecting an overall result. Data from 2033 spending units interviewed in early 1964 in the 1963 Survey of Consumer Finances (29) were used. The following explanatory variables were employed:

Number of subgroups
Stage in the family life cycle ..... 10
Education of the head of the unit ..... 6
Age of head ..... 6
Size of place of residence ..... 6
Race ..... 4
Income change over previous year ..... 4
Region of the country ..... 4

The twenty final groups accounted for half the total variance. The standard cutoff criteria which allowed any split which reduced the error by $1 / 2$ per cent allowed one final (omitted) split which formed groups of one and four cases respectively. It is quite clear from Chart 11 that the income of spending units depends mostly upon whether they are married, educated, middle aged, live outside the South, and live in metropolitan areas. The first split points to those "married and not retired" which means at least one earner and in many cases two. The other group are handicapped by being extremely young or old, having children but no spouse, or (and here the causation may go the other way) by having no family responsibilities.

We may summarize the next split on education by saying that the group with advantages is depressed only by very low education, but the
disadvantaged group as to family situation is redeemed only if the head is a college graduate.

Following up the top set of branches, we note that a combination of advantages cumulate into substantial incomes. The higher income among one group of college drop-outs than among college graduates may be explained by their age (45-54), which means that they dropped out during the great depression. This may be a chance fluctuation, however, since a reverse effect is apparent among the same cohort living in small towns or rural areas, as well as among other age groups. This problem could be pursued further by a deviant case analysis with the object of determining what factor (s) are common to members of each of these two apparently contradictory groups.

As one way of assessing the stability of the resulting subgroupings, an analysis was made of spending unit disposable income for three separate Surveys of Consumer Finances covering incomes for the years 1952, 1957 and 1962. In addition, the 1957 subgroups were formed with the 1962 data to see how well they could explain data from which they had not been derived.

In different years, there was a good deal of agreement as to which predictors accounted for most of the reduction in the unexplained sum of squares, except that age, education, and stage in the life cycle increased greatly in explanatory power over time (but the last resulted from a more detailed coding of life cycle). It turned out (30) that there was a real change toward a greater earning payoff from education that took place during the period (see Table 13).

The order in which the branching took place varied from year to year. The reason is probably that there are several alternative ways to achieve roughly the same subgroups--one can separate the college graduates, then the middle aged among the college graduates, or start by selecting the middle aged, then separate the college graduates. Sampling variability may well be influencing which of two almost equally good predictors will be used.

This means that the proper focus in investigating sampling stabi1ity should be on the composition of the final groups, the interpretation

Table 13
AID ANALYSIS OF SPENDING UNIT DISPOSABLE INCOME
--1952, 1957, 1962

| Predictors | Reduction inTSS (I)/TSS (T) |  |  | Gross Beta Coefficients ${ }^{2}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1952 | 1957 | 1962 | 1952 | 1957 | 1962 |
| Place of residence | . 034 | . 029 | . 042 | . 042 | . 033 | . 032 |
| Age of head | . 029 | . 034 | . 059 | . 064 | . 081 | . 124 |
| Education of head | . 124 | . 114 | . 171 | . 127 | . 126 | . 179 |
| Race | . 005 | . 000 | . 008 | . 030 | . 033 | . 034 |
| Region | . 021 | . 000 | . 016 | . 016 | . 003 | . 003 |
| Life cycle | . 095 | . 128 | . 201 | . 107 | . 135 | . 197 |
| Income change | . 021 | . 010 | . 007 | . 018 | . 028 | . 036 |
| $\mathrm{R}^{2}$ | . 329 | . 315 | . 504 |  |  |  |

Source: ISR Study 719, MTR's 28-30
of the combinations of factors (pedigree) they represent, and on the explanatory power of the predictors at different stages in the tree rather than on the paths. It also means that even the total explanatory power assigned to various factors is stable only in a rough sense.

One may also compare the total explanatory power of the 1957-derived subgroups for 1962 data. The proportions of total sum of squares accounted for are presented in Table 14.

Table 14
AID ANALYSIS OF SPENDING UNIT DISPOSABLE INCOME, 1957, 1962

|  | $\mathrm{R}^{2}$ |
| :--- | :--- |
| 1957 tree, 1957 data | .315 |
| 1957 tree, 1962 data | .366 |
| 1962 tree, 1962 data | .504 |

The increase in explanatory power of some of the factors over time makes it necessary to qualify any conclusions, but it is clear that the 1957 tree is not so good in 1962 as one based on the 1962 data, yet neither is it so inferior that one would regard it as an unstable, fortuitous breakdown of no use for prediction.

Another experiment involved split-half samples carefully designed to take account of the original stratification. Three different split halves were run on hours worked and three on hourly earnings. Again, while the way in which they were developed differed; the final groups were reasonably similar, and the ranking of factors by importance reasonably comparable. The proportions of unexplained sum of squares were much higher for the split halves, because the cut-off rules are less stringent with smaller samples. In other words, explaining $1 / 2$ per cent of the total sum of squares of a smaller sample, using the same possible subclasses, leads to more subdividing and hence explains more of the variance.

CHART 11
SPENDING UNIT DISPOSABLE INCOME -- 1962 (BASED ON CHARACTERISTICS OF SPENDING UNIT HEADS)

Section 3.9 Two Year Saving as Per Cent of Income
So far the initial analyses have been by multiple regression. The AID analysis sought to discover new things about the data not revealed by the regression. As an example of a more appropriate process, we turn now to a case where the AID analysis was used to determine which new (interaction) variables should be created and used in a regression analysis. The dependent variable was two-year saving as a percentage of two-year income from a panel study (34).

Earlier analyses had been run on the first version of the program with saving rate, discretionary saving rate, and an index based on an ordered series of saving rate classes as dependent variables, but the large number of classifications with 8,9 , or 10 subclasses combined with the relatively small sample provided many fortuitous combinations. The tree presented here (see Chart 12) made use of the option to maintain the order of subclasses for nine of the twenty-one predictors. It still tends to use predictors with too many subclasses, and combines clearly exogenous factors with some which might be results as well as causes. The variables used are listed in Table 15.

Sixteen of the twenty-one factors were used to form twenty-seven groups that accounted for 32 per cent of the total sum of squares. The AID analysis split first on home ownership (treating homes worth less than $\$ 2500$ as not owning), then split both branches on whether the head was a self-employed businessman or farmer. Some other groups were split off from each of the nonentrepreneurial branches, notably low saving groups who had spent a lot on consumer investment items (cars, durables, additions and repairs), but this could be regarded as partly circular, i.e., as a decision to buy durables rather than save. The most important subsequent split was one which used initial assets, but split home owners and nonowners at a different level and revealed that owners with high initial assets saved more than other owners, while nonowners with some initial assets saved less than other nonowners.

This threw light on an ancient discussion about the effects of assets on spending and saving. Some economists had argued that assets facilitated spending, burning a hole in the man's pocket. Others said that those with assets were motivated to save and would persist in this
behavior. Our analysis seemed to say that the best way to separate those with a persistent tendency to save from those with a high marginal propensity to consume, was to separate home owners from others.

The rest of the tree is complex and shows problems that arise when complex variables created ad hoc are introduced instead of built by the analysis from their components. Some of the later splits involve very small numbers of cases and have been recombined.

A neater format for presenting data, and a tightening up of this notion, required developing a set of dummy variables and putting them into a multiple regression, to assure that these relations could hold their own with other variables against a charge of spurious correlation. Several others of the sets of subgroups in the regression were developed deductively, others were unidimensional, and some had a long mixed history of development (stage in the family life cycle). The factors included and their partial beta coefficients are presented in Table 16.

To have put in "whether a self-employed businessman or farmer" as a separate dummy variable would have been to assume that home-ownership and assets affected the saving of these people too. A glance at the AID tree will reveal that this is not the case.

Needless to say, no significance tests should be applied to variables derived from a second analysis of the same set of data, and there is even a question about those derived by analysis of similar sets. On the other hand the five subgroups have reasonable and meaningful differences. They also serve the purpose of controlling on some factors (removing unwanted "noise") in a test of other factors in the regression. The unadjusted saving ratios, and the ratios adjusted by regression are given in Table 17 below. (Regression adjustment means adding the constant term to the dummy variable regression coefficient, the result being what the saving ratio of that group would be if it were like the whole population in its distribution on all the other variables.)


Table 15
VARIABLES USED TO PREDICT TWO-YEAR SAVING AS A PER CENT OF INCOME

|  | Subclass order free or monotonic | Number of subclasses |
| :---: | :---: | :---: |
| Stage in family life cycle | F | 10 |
| Number of people in the spending unit | M | 9 |
| Occupation | F | 5 |
| Age of head | M | 7 |
| How long lived in this residence | M | 8 |
| Bracket value of house | M | 10 |
| Home ownership status | F | 6 |
| Education of head | M | 6 |
| Anticipated course of income over next ten years | F | 6 |
| Course of income over past ten years | F | 9 |
| Level of optimism in early 1961 | F | 3 |
| Level of optimism in early 1960 | F | 3 |
| Two-year expenditures on cars, durables and additions and repairs as per cent of two-year income (bracket) | M | 10 |
| Two-year income (bracket) | M | - 11 |
| Size of place (city size) | M | 6 |
| Expected income change in 1962 | F | 4 |
| Income change from 1958 to 1959 (memory) | F | 4 |
| Sources of income change 1958-1959 | F | 10 |
| Sources of income change 1959-1960 | F | 10 |
| Pattern of past and expected income change | F | 5 |
| Total assets in early 1960 (bracket) | M | 7 |

Table 16
RELATIVE IMPORTANCE OF 14 SETS OF DUMMY VARIABLES
IN A MULTIPLE REGRESSION

$$
(\mathrm{N}=1001)
$$

| Characteristic | ```Relative importance partial \beta``` | Number of subgroups |
| :---: | :---: | :---: |
| Occupation-house-value-assets | . 060 | 5 |
| Stage in family life cycle | . 022 | 9 |
| Two year income | . 021 | 10 |
| Pattern of past and expected income change | . 015 | 5 |
| Age of head of unit | . 011 | 6 |
| Sources of income change 1960-1961 | . 010 | 10 |
| Sources of income change 1959-60 | . 010 | 10 |
| Size of place of residence (city size) | . 009 | 6 |
| Changes in optimism | . 009 | 6 |
| Anticipated income change 1961-1962 | . 006 | 4 |
| Years lived at present address | . 005 | 6 |
| Educational attainment of head | . 004 | 6 |
| Course of income over past ten years | . 003 | 9 |
| Anticipated course of income over next ten years | . 002 | 6 |

Table 17
UNADJUSTED AND ADJUSTED SAVING RATIOS
$\left.\begin{array}{lll}\hline \hline & & \begin{array}{c}\text { Two year saving } \\ \text { as per cent }\end{array} \\ \text { Characteristic } \\ \text { of two year income }\end{array}\right)$

Source: ISR Project 715, Deck T

Section 3.10 A Two-Stage Analysis: Hours Worked-Head
Charts 13 and 14 provide another illustration of a two-stage analysis. The data are taken from a national sample of spending units (24). The dependent variable is the number of hours worked by the head of the spending unit during the year. The analysis is performed on only those units where the head worked during the year.

The mean of the original distribution analyzed is 2092 hours. Its standard deviation is 797.

Predictors were divided into two parts, those felt to be early, or basic in a causal chain which might explain variation in hours worked, and those which were regarded as probably having later, more direct effects (see Table 18). The residuals from the first analysis were computed and were used as input to the second stage.

The interpretation of the stage one tree is straightforward. However, several things should be noted. In the split of group 3 into groups 12 and 13, those aged 75 and over are put together with the age 25-54 group. There are only six such observations. The split of group 7 into 8 and 9 is somewhat unexpected. Why should "having grown up on a farm outside the deep South" lead to long hours of work?

One plausible interpretation is the upward push of habits of work associated with farm background, uninhibited by the depressing effects of southern rural background (or associated race).

Notice that all the other splits in this tree involve separating off a group inhibited from working by some handicap, none of these groups being split again. The inference is that such handicaps are alternatives, any one being sufficient to keep a person from a full year's work.

This analysis accounted for sixteen per cent of the variation.
The second phase of the analysis included a large number of predictors, including some of those already used in the first phase. Four of them were constrained (monotonic). The dependent variable was the residual from the first phase. For each input observation, a large positive residual indicates that the dependent variable was larger than its predicted value. The mean of the dependent variable
for this run was -5 (the departure from zero is due to truncation and rounding error). Its standard deviation was 732 hours. This second analysis explained 22 per cent of the variation in the residuals.

Chart 14 is quite plausible and meaningful. The most important factors reflect not motivations of the usual economic sort, but constraints, such as working for others (who set the hours of work) or being unemployed. After these effects are at least partially accounted for, it is clear that lower hourly earnings are associated with longer hours of work. Finally among a low-wage, self-employed group, those who migrated out of the South appear to work the longest hours of all. If these are people with ambition but not a great deal of education, many of them still farmers, the result makes sense.

Unemployment experience can be thought of as not so much a cause for shorter hours, but as a joint result; both unemployment and short hours resulting from lack of basic skills or living in a labor surplus area. This serves as an explanation as to why some people work more than others, even after the main effects of age and education, etc., have been removed.

The tree was truncated by omitting two further splits using stage in the family life cycle, and selecting combinations of that combination which were difficult to interpret. This provides one more example of the need to restrict the explanatory factors to one dimension each.

CHART 13
ilOURS WORKED -- HEAD
(EXCLUDES SU HEADS WHO DIDN'T WORK)


SOURCE: ISR Study 678
Deck 35, MTR 50

* Plus six cases 75 or older


Table 18
VARIABLES USED IN THE ANALYSIS OF HOURS WORKED IN 1959 BY SPENDING UNIT HEAD

| Number of <br> subgroups | Monotonic <br> or free |
| :--- | :--- | Description of Classification

## First Stage

| 4 | M | Physical condition, whether or not a physical <br> disability is reported |
| :--- | :--- | :--- |
| 8 | M | Education of heads |
| 2 | F | Race |
| 7 | F | Age |
| 8 | F | Sex, marital status, and number of children |
| 4 | F | Major religious affiliation (Protestants <br> separated into Fundamentalist and <br> non-Fundamentalist) |
| 4 | F | Index of need for achievement (three groups <br> pIus not ascertained) |
| 6 | F | Where heads grew up-deep South, rest of U.S., <br> abroad, and whether on farm or not |

## Second Stage

| 9 | M | Wage rate of heads |
| :---: | :---: | :---: |
| 6 | M | Number of states lived in, and whether heads ever lived more than 100 miles from present residence |
| 3 | M | Whether heads or fathers grew up in a foreign country |
| 10 | F | Occupation of heads |
| 3 | F | Whether heads are self-employed, or supervise others, or neither |
| 8 | F | Frequency of unemployment of heads |
| 7 | F | Religion and frequency of attendance |
| 7 | F | Index of need for achievement and belief that hard work is more important than luck and help from friends |
| 2 | F | Race |
| 9 | F | Stage in the family life cycle (married, wife under or over 45, pre-school children, school children) |
| 7 | F | Difference in education between heads and their wives |
| 5 | F | Where heads grew up and where they live--urban rural migration |
| 6 | F | Where heads grew up (deep South?) and now live --north-south migration |
| 6 | F | Unemployment in the area--U.S.B.E.S. ratings |
| 4 | F | Plans to help parents or to send children to college |
| 6 | M | Size of place (city) |
| 4 | F | Difference in education between heads and their fathers |
| 2 | F | Sex of heads |

## INTERPRETATION AND ANALYSIS STRATEGY

## Section 4.1

## Structure of the Trees

The analyses that have been presented show a series of characteristic tree patterns. Each tree has sections that can be described as a combination of two configurations, based on the useful convention of showing the group with the highest mean as the uppermost branch. One may be termed a trunk-twig structure, the other a trunk-branch structure.

The trunk-twig structure is a main branch from which small groups are split off from the main branch and are not themselves split again. This may take three forms, top-termination, bottom termination, and alternating termination. The top-termination structure may be termed an "alternative advantage" model. Group B consists of those observations possessing the "advantage" represented by that characteristic which split group A into groups B and C. Once group B has been established, it cannot be split further by the program.

The bottom-determination structure may be termed an "alternative disadvantaget model, and is analogous. The possession of any one of a number of characteristics is enough to prevent an observation from achieving a high value on the dependent variable.

The interpretation of the alternating termination configuration is similar. In all three types, the interpretation to be made depends on the characteristics of the final groups themselves, especially on the number of observations in the group, its variance, and whether or not there existed predictor variables which "almost worked" in the attempt made by the program to split it.

Another property of the tree is its symmetry or nonsymmetry in terms of the extent to which the same variables are used in the splits


TOP TERMINATION


BOTTOM TERMINATION


ALTERNATING TERMINATION
on the various trunks. Nonsymmetry implies interaction, i.e., effects of combinations of factors. If a variable is used on one of the trunks, and if it shows no actual or potential utility in reducing predictive error in another trunk, then there is clear evidence of an interaction effect between that variable and those used in the preceding splits. The utility of a predictor in reducing predictive error is evaluated by statistic ( $\left.\mathrm{BSS}_{\mathrm{mpr}} / \mathrm{TSS}\right)_{i}$ for each predictor at each branch in the tree. This output is produced by the program and represents the proportion of the variation in the group to which the predictor is being applied that would be explained if it were used in a binary split of that group.

Trees may, of course, be symmetrical with respect to the way in which top-termination, bottom-termination and alternating-termination configurations appear in the main trunks.

The trunk-branch structure is usually typical of the first few splits of any tree. In this case, each group produced by a split is further subdivided.


TRUNK-BRANCH STRUCTURE

Some of the early groups may remain unsplit. If this is so, then the most important aspect of the interpretation of this structure has to do with the fact that there remains within-group variation which can be explained. At each step, the analytic question that should be asked
is, "What are the reasons why there is as much variation in each of the groups as there is?" This question will be discussed below in more detail.

A further property of each tree is the number of final groups that result from the analysis. This is, of course, a function of the input sample size, the statistical properties of the algorithm, and the relationships between the characteristics of the predictor variables and the dependent variable.

Based on the present characteristics of the algorithm, we can distinguish three types of final groups: small groups, explained groups, and unexplainable groups. A small group is one containing too few observations to warrant an attempt to split it. An explained group is over this minimum size, but has too little variation in it (less than, say, 2 per cent of the original variation) to warrant an attempted split. An unexplainable group is sufficiently large and spread out, but no variable in the analysis is useful in reducing the unexplained variation contained within it. Each tree will generally have some of each of the three types. But the total number of final groups is heavily dependent on the rules used to stop the splitting process.

Section 4.2

## The Rules for Stopping

What are the statistical considerations behind the choice of rules as to where to stop the splitting process? Just as there is no point in making any but the most important split at each stage--allowing other variables a later chance--so there is no point in making splits which are likely to be heavily influenced by sampling error.

It seems unreasonable to apply ordinary statistical tests at each split; that is, to insist that the split be a statistically significant difference between the two means. It is the best of a large number of possible splits at each stage. Even ignoring the re-ordering of subclasses, there are $\mathcal{C}_{\mathrm{x}}-1$ possible splits for each predictor at each stage (less some that have been eliminated because of previous splits), and the deductive logic associated with these tests does not apply.

The primary test is one of importance, i.e., the reduction in the error sum of squares. This is kept from being too arbitrary by expressing it as a per cent of the original total sum of squares. This is equivalent to saying that if there is a great deal of variation, the two new group means must be more disparate than if there is less variation. The use of error reduction also handles the problem of different numbers of observations in the two new groups, since the greater the disparity between group sizes ( $N^{\prime}$ s), the larger the difference between the means has to be to produce the same between-group sum of squares.

A separate test of significance, in addition to the test of importance, might be desirable in spite of the difficulties about degrees of freedom if there are likely to be splits which are not significant even on the boldest assumption, but which produce substantial error reductions. This tends to be true with skewed distributions and very small numbers of observations in a number of subgroups. But when this happens, there are serious problems no matter what multivariate technique is used.

The standard error of the difference between two means is:

$$
\begin{equation*}
\sigma_{\bar{Y}_{1}}-\overline{\mathrm{Y}}_{2}=\sqrt{\frac{\sigma_{1}^{2}}{\mathrm{~N}_{1}}+\frac{\sigma_{1}^{2}}{\mathrm{~N}_{2}}} \tag{4.2.1}
\end{equation*}
$$

If, in each split, we make the strong assumptions that the resulting two groups are roughly the same size and that their standard deviations are the same, then the standard deviation of the difference is approximately:

$$
\begin{equation*}
\sigma_{\overline{\mathrm{Y}}_{1}}-\overline{\mathrm{Y}}_{2}=\frac{\sqrt{2} \sigma}{\sqrt{\mathrm{~N}_{1}+\mathrm{N}_{2}}}=\frac{\sqrt{2} \sigma}{\sqrt{\mathrm{~N}_{0}}} \tag{4.2.2}
\end{equation*}
$$

where: $\sigma_{1}=\sigma_{2} \leqq \sigma$
and: $\quad 2 N_{1}=2 N_{2}=N_{0}$, the size of the group being split.

Under the assumptions, this quantity differs from split to split, depending on the values of $\sigma$ and $N_{0}$. But in practice, $\sigma$ typically varies from split to split much less than $N_{0}$. It was for this reason that it was decided to add a cut-off criterion based on the size of the group to be split.

If a difference between two means is to be significant (say, more than twice its standard error), then it is required that:

$$
\begin{equation*}
\left(\bar{Y}_{1}-\bar{Y}_{2}\right)>\frac{2 \sqrt{2} \sigma}{N_{0}} \tag{4.2.3}
\end{equation*}
$$

and

$$
\begin{equation*}
\sqrt{\mathrm{N}_{0}}>\frac{2 \sqrt{2} \sigma}{\left(\overline{\mathrm{Y}}_{1}-\overline{\mathrm{Y}}_{2}\right)} \tag{4.2.4}
\end{equation*}
$$

hence,

$$
\mathrm{N}_{0}>\frac{(2 \sqrt{2} \sigma)^{2}}{\left(\overline{\mathrm{Y}}_{1}-\overline{\mathrm{Y}}_{2}\right)^{2}}
$$

and

$$
\begin{equation*}
N_{0}>\frac{8 \sigma^{2}}{\left(\bar{Y}_{1}-\bar{Y}_{2}\right)^{2}} \tag{4.2.5}
\end{equation*}
$$

Thus, one might not wish to make a split which was not "significant" even under these extremely lenient assumptions.

This implies that the minimum group size to be eligible for split- . ting really should depend on the standard deviation of the dependent variable, and on the size of the difference between the means of the two prospective new groups. In other words, in developing a rule about minimum group size, we should also pay some attention to the variance of the dependent variable.

But there is also the problem of the "number of things tried," which is relevant to the problem of fortuitous splits. The probability of this happening must be proportional to the number of possible splits at each step, since if we had enough classes available in the predictors, and a sufficient number of such predictors, we should be able to reduce the unexplained variation by half with each split. Thus, a term such as:

$$
\begin{equation*}
K=\sum_{i=1}^{N P}\left(c_{i}-1\right) \tag{4.2.6}
\end{equation*}
$$

which is the total number of classes for each predictor (minus one), summed over all predictors, should also be taken into consideration. We note, however, that even this ignores the re-ordering of the classes during the partition scan.

In other words, for a group of any given size $N_{0}$, the larger the number of predictors and the more classes per predictor, the larger is the chance of finding a (fortuitous) split that is "important," but not "significant," particularly if one raised the significance levels to fit the situation (and the assumptions described by formulas 4.2.14.2.5 provide a situation which is one of the most powerful alternatives).

Clearly, there are a number of interesting problems in mathematical statistics raised here, the solution of which might lead to clearer rules about how to set the four cut-off criteria, the total number of final groups, the minimum interior sum of squares for a group to be eligible for splitting, the minimum number of cases for a group to be eligible for splitting, and the minimum between group sum of squares if a split is to be made at all.

Figure 1 provides an example of the relationship between the size of the split reducibility criterion (the program input parameter P2) and the number of final groups. The original analysis was run with this criterion set at . 002 . There were 13 predictors with a total of 70 classes and a sample size of 2569 . The minimum group size rule was not used. The eligibility criterion $\mathrm{P} 1=.02 \mathrm{was}$ used. The tree was then "pruned"; that is, it was determined how many final groups would have resulted if the reducibility criterion had been set at progressively higher levels. The resulting curve is a hyperbola which becomes asymptotic along the reducibility ( R ) axis at one, since there must be at least one final group, and which becomes asymptotic along the $G$ (number of final groups) axis at about .002. The maximum possible number of final groups is, of course, $N$, the input sample size. The curve:

$$
\begin{equation*}
G=\frac{1}{\left(\sqrt{\frac{K-P}{N}}\right)_{R}} \tag{4.2.7}
\end{equation*}
$$

where $K$ is the total number of classes over all predictors, where $P$ is the number of predictors, where N is the total number of input observations, and where $R$ is the split reducibility criterion,
provides a reasonably good fit to the points observed. Further, if we plot $G$ against $X$, where $X=(K-P) / \sqrt{N}$, with $R$ held constant at .005, we find that (approximately) $G=12 X+1$, so that for most cases, the relationship between $G$ and $X$ as defined is linear. Four cases do not fit this general model. All are truncated or badly skewed distributions.

Other analyses of this type have not been performed. This family of curves is suggested as an example of the lines along which some further investigation is needed.

All that can be stated here are some general rules which apply to the range of sample sizes, kinds of data and numbers of predictors which we have used.

FIGURE I
NUMBER OF FINAL GROUPS AS A FUNCTION OF $\sqrt{N}$,


SOURCE ISR STUDY 678 DECK 35
719; MTR II

1. For typical survey data a minimum group size of 25 seems reasonable, since one hardly ever puts much credence in two subgroups whose combined N's add to 25 or less, however different their means. Any group $i$ with $N_{i}<25$ should not be split. With other kinds of data with less error, however, a smaller number might be appropriate.
2. An eligibility rule that a group must contain at least two per cent of the total original sum of squares if it is to be considered eligible for splitting has the disadvantage that the program, as currently written, presents no data on the distributions of the predictors over that group. Indeed, this rule should be regarded as the least important of the four and kept low enough so that it is seldom, if ever, used. The minimum group size is more meaningful, and failure to find an acceptable split even more so. With minimum group size set at 25 , and a reducibility criterion of .005 of the original total sum of squares, an eligibility limit of .015 seems to be low enough to assure that the other rules predominate.
3. The maximum number of groups should be regarded as a safety rule to cut off the program if something goes wrong, i.e., if the other rules were improperly set. There may be instances, however, where one wants only the best ten or twenty groups for some reason, such as developing procedures for assigning missing data, or developing a single new variable out of several raw variables.
4. The split reducibility criterion appears to be the crucial one to set; that is, the relative size of the between group sum of squares from a split which is necessary to allow that split to be made (after the best available one has been selected). The standard is like the one per cent or five per cent rule for significance tests. It is somewhat arbitrary.

Our experience has been that with K less than 100 (formula 4.2.6) and samples of 2,000 to 3,000 , and with a dependent variable that is not too badly skewed, the resulting trees seem manageable and interpretable with a requirement for error reduction of . 005 . With more predictors, or smaller samples, the criterion should be raised.

The skewness of the dependent variable also influences the number and type of the final groups produced. For example, in one case with a very large number of predictors and a sample of 1000 , we produced a reasonable number of final groups using .005 as a reducibility criterion. But when we omitted 38 extreme cases (which accounted for 53 per cent of the total sum of squares), the same rules produced twice as many groups from the remainder.

These problems are not so serious as they might seem, since it is always possible to truncate (prune the tree) either for higher minimum group sizes or for a higher minimum split reducibility criterion. It is not possible to truncate on the basis of the size of the trunk-twig subgroups, since once one is split off, the remaining trunk is affected. Hence, if the dependent variable is skewed and a number of groups consisting of one or two observations are split off, these twigs cannot be pruned. In this case, the extreme cases should be removed, explained separately, and the analysis re-run without them, or else the dependent variable should be transformed into a somewhat more normally distributed form, perhaps using logarithms.

An added reason for using the split reducibility criterion rather than the others to do the real work of cutting off is that in this case all predictors are tried and the results printed out for each final group as well as the intermediate ones.

The analyst must decide whether each split shall be regarded as real or as containing fortuitous elements which should cause it to be disregarded. We have presented a rationale for setting the input parameters in such a fashion so as to minimize the probability of the occurrence of splits which are important (in the sense that they reduce the unexplained variation by a large amount), but not significant (in the sense that they could quite reasonab1y have occurred by chance).

We have investigated the sampling stability of the procedure in a limited fashion by using split-half techniques and by using the tree produced from one sample to predict values of the same dependent variable in another sample. Though the results seem quite encouraging, much more work needs to be done in this area.

In addition, we have also noted that the number of classes in the predictors are factors which contribute to the probability of a fortuitous split occurring, and that whether or not any given predictor has had its ordering constrained (is assumed to be monotonic) will affect the probability of its being involved in a fortuitous split. We have indicated a two-part rule for minimizing the effects of a large number of classes and the increased probability of a fortuitous split when a predictor is unconstrained.
a. Predictors which have a natural rank ordering to their classifications should be constrained to that ordering during the partitioning process, except where the possibility of a U-shaped or inverted U-shaped relationship between that variable and the dependent variable is suspected, in which case adjoining classes should be combined to form a maximum of five classes and the variable left unconstrained.
b. Un-ordered predictors should not have more than five or six classes, and should be left unconstrained.

We now turn to a description of techniques for displaying the results and to the problems of interpreting the behavior of the variables in the trees.

A number of techniques can be used for summarizing and displaying the data produced by the AID (2) program. A number of these have been presented in previous sections. They may be described as follows:

1. The tree itself. A useful convention is to represent those groups with the higher means on the upper branches of the tree. A group may conveniently be represented as a box containing a short description of the predictor classes used in that particular split and which are included in the group, together with its mean, and standard deviation. We have included the N , or group size only on the final groups. A useful convention is to display the per cent of sample on each line leading to a box (see Chart 12). For convenience, an asterisk or other indicator may be used to mark a final group on which an attempted split was made, but which failed; that is, an unexplainable group.
2. The statistic (BSS/TSS) ${ }_{i}$ can be examined for each predictor over each group created during the partitioning process (see Table 1) with suitable indicators to mark those variables used in the splits, other variables which were almost effective enough to be used, split fail attempts, and terminal groups.
3. For a rather gross, overall description of the behavior of the variables in the tree, a tabulation of the reduction in unexplained sum of squares attributable to the splits using $\beta^{2}$ for each predictor appears to be useful (see Table 14). This could also contain the statistic $B^{2}$, or gross effect of that variable if it were used in a one-way analysis of variance with all its classification detail, but without considering the effects of the other variables. This table also facilitates comparison with multiple regression statistics.
4. A detailed analysis of the behavior of one predictor is facilitated by the construction of a table (see Table 9) which shows all of the various classes of that predictor, and mean values of the dependent variable over these classes for the total input group,
and for the splits in which this predictor was involved. This table could also be developed for each subgroup occurring in the tree.
5. A description of the final groups listed in rank order on their means. This provides a summarization useful for presentation (see Table 11).
6. Frequency distributions of each of the predictors for each of the final groups provide additional information about the behavior of the variables. If residuals are punched from the program, obtaining such frequency distributions for variables not in the analysis is straightforward, and provides a method for investigating the extent to which variables have substituted for each other in the analysis. Distributions for the predictors used can also be produced by running a second-stage AID analysis using the residuals as the dependent variable.

These summarizations provide a number of devices for collecting the large amount of information produced by the program and organizing it in a fashion which facilitates the decision making process that constitutes the analysis.

Section 4.4 The Behavior of the Variables in the Trees
The analysis of the behavior of the predictors and their relationship to the dependent variable during the partitioning process can be approached through a series of questions, asked with reference to each partition.

## Chance Factors

The first question is, "Given the minimum group size rule, split reducibility rule and split eligibility rule used, what is the likelihood that this split occurred by chance?" This problem may still occur even if the above-suggested rules have been used for minimizing the probability of its happening. If a variable actually used in the split is the only one which shows up as important, according to the criteria used, then the probability of its predictive power being based largely on sampling variability is relatively slight, unless it is an unconstrained variable with a large number of classes. When several variables are almost equally good as predictors, in any given split, then the likelihood is greater that sampling variability has had a hand in selecting one, rather than another, as that variable to be actually used in the split. The (BSS/TSS) ${ }_{i}$ tabulation (display method 2, above) provides a guard against basing an interpretation only on those variables actually based in the partition process, since the explanatory power of the unused predictors is presented in all its detail.

The overall structure of the tree provides a clue as to the probability that sampling variability is operating together with a skewed distribution.

In the case where the dependent variable is badly skewed and has a tail extending toward the right (positive skewness), a top-terminating trunk-twig structure is likely to appear in several main branches of the tree. These terminal groups will have large, positive means, and will contain few (1-5) observations. Typically, they will result from splits on several different variables. Sooner or later the program will find some predictor which enables it to split out these extreme cases from the group in which they happen to be.

As we have mentioned previously, a careful re-reading of interviews may turn up a variable, certain values of which most of these extreme cases will have in common. This variable may then be inserted into a subsequent analysis. One may be reasonably confịdent that these observations will then be placed together in one group via a split on this variable. Good strategy would, therefore, dictate a preliminary investigation of the skewness of the dependent variable before the main analysis starts.

One might construct a dummy variable which has the value one if an observation is out in the skew tail and zero if it is not. A preliminary AID analysis, using this as the dependent variable, together with the predictors to be used in the main analysis will provide information as to which classes of the sample are out in the tail, rather than being in the main body of the distribution. It may be that one set of variables will be found optimal to explain being out on the tail of a distribution. Another set might prove best for explaining overall variation or variation in the main body of the destruction. This possibility would, of course, be of considerable theoretical importance.

Of course this technique need not be confined to observations out in a skew tail of the dependent variable distribution. For some analytic purposes it may be desirable to use this technique to determine what combination of variables are associated with an observation's being, say, in the second quartile of the distribution, or less than some specified value.

It should be noted that a variable which is not skewed in the total sample, may become skewed during the partitioning process. This cannot be caught in advance. Hence even when a preliminary investigation of skewness has been made, the analyst should be on his guard for the appearance of this particular trunk-twig structure (see Section 3.5). A bottom-terminating trunk-twig structure with small terminal groups would provide a signal for negative skewness.

## Conceptualization Problems

A second question that should be asked is, "Does this split reflect conceptualization problems in applying the framework of predictor variables to the sample, or sections of it?" A number of interpretation problems in the trees may stem from measurement or coding errors, or from the use of variables that were designed for other statistical purposes. This technique is at its best when the predictors have a clear, uni-dimensional reference. We have presented one example of a conceptual problem that looked, initially, like a somewhat contradictory finding, until coding decisions were uncovered which appeared to misclassify uneducated people living on the fringes of cities of 50,000 and over, with respect to the rural or urban nature of their surroundings. Indices having several components also tend to behave in a somewhat peculiar fashion. Presumably, this is because the items in these indexes, though related both theoretically and statistically, may affect the dependent variable in different ways, particularly if some of them interact with other variables in the tree and others do not. Splits involving such variables may or may not "make sense." See Coombs (31) for a thorough discussion of scaling problems.

Perhaps the most important point to be made here is that problems like these are often revealed only by large standard errors that may accompany a multiple regression analysis. They tend to stand out quite clearly in the tree display of the AID results.

## Substitution of Variables

A further question which should be asked with reference to any given split is, "Are there competing predictors correlated with the one actually used in the split? If so, does their explanatory power increase, decrease, or stay the same in subsequent splits?" The logic to be employed here is developed extensively by Hyman (2) in his discussion of spuriousness, and in his presentation of M- and P-type elaboration. He presents a formalization of the logic of examining the relationship between two variables when a third factor is introduced. The two factors under examination are referred to as $x$ and $y$, and
the third is called $t$. In our notation, $x$ is the variable used to split group $i$ into groups $j$ and $k ; y$ is the dependent variable, and $t$ is multiple and consists of each of the other predictors in the analysis. We are interested in the relationship between variable $t$ and variable $y$, as represented by the statistics (BSS/TSS) ${ }_{i}$, (BSS/TSS) ${ }_{j}$ and (BSS/TSS ${ }_{k}$ for each predictor $t$. If, in addition, we consider whether or not there is a logical, theoretical justification for a correlation between $x$ and $t$, and if so, whether $x$ can be conceptualized as antecedent to $t$ in a causal chain, we have a systematic application of the analysis strategies of:

1. Interpretation ( $t$ is an intervening variable)
2. Explanation ( $t$ is antecedent to $\mathbf{x}$ and is logically related to it)
3. Control for spuriousness ( $t$ is antecedent to $x$ and cannot be related logically to it)
4. Specification ( $t$ is neither antecedent to $x$ nor subsequent to it, but is logically related. Here $x$ is a circumstance that affects the extent to which $t$ is related to $y$.)

The reader is referred to Hyman (2) and to Blalock (32) for the details of the logic.

We note that we have reverted to a form of the analysis question, "Other things being equal, how does $x$ affect $y$ ?" but in a somewhat different form. We now have the question, "When we extract variation associated with predictor $x$, how do the relationships between $t_{1}$, $t_{2}, \ldots, t_{p}$ and $y$ change?"

In providing an answer to this question that is meaningful, the question of the substitutability of variables in the analysis must be taken into consideration. This is the problem of intercorrelations between the predictors. Numerous examples may be seen in the trees. The variable "number of wage earners in the family" may really be serving to split off some old, retired people. The variable "pattern of income change" may really be splitting off people who are not in the labor force, i.e., old and retired. It is impossible here to consider all the problems associated with the relationship between a variable and the concept(s) it purports to represent, but a few points should be emphasized.

Some intercorrelations are built into the data by the coding process. Other high correlations may result because two predictors may themselves be the results of a third factor which may or may not be represented in the analysis by a variable. Still others are there because things go together in the real world. But it is on exactly this structure of relations that we are trying to get a grip. What is required is a strategy for minimizing the interpretation problems.

One way to deal with this is to put in the most clearly exogenous, most orthogonal and uni-dimensioned variables into a first-stage analysis, together with a relatively high reducibility criterion and fairly large minimum group size, and then use the richer matrix of predictors for an analysis of the residuals. Where a tight test is desired as to whether a variable which is of considerable theoretic importance has effects, this variable may be held out of the first-stage analysis and entered in the second stage to see whether it enables the explanation of residual variance. If a low eligibility criterion is used, the present algorithm will make a final sweep over all the final groups before dropping them from consideration, thus providing information on how all of the predictors are distributed within each group. (The present version of the program will not provide this, however, if the final group size $\left(N_{i}\right)$ is under the specified minimum.) These distributions can be used to provide information as to whether the group occupies its present place because of its actual pedigree or because of some other factor (s) correlated with the ones used to form it.

Moreover, it would certainly be desirable to obtain information on the zero-order correlations among the predictors in the sample. Since they are classifications, this is not easy. A complete set of bivariate frequency distributions provides a general impression. Further improvements in the algorithm itself should provide for a satisfactory method of computing the intercorrelation matrix of predictors at each branch of the tree.

If there are some variables which, because of high intercorrelations, or low logical priorities, must be put into a second-stage analysis, one will not know (and has decided not to ask) what their
influence would have been in the formation of the first-stage groups. The second stage, however, will show whether or not their influence on the dependent variable has already been accounted for. Re-introducing the first-stage variables into the second stage will also provide an answer to the question of whether there is a small, but universal, effect across all groups which will appear when they are pooled for the residual analysis.

In some cases, the first-stage analysis will identify groups which are clearly constrained in some special way, and explained so clearly that they really should be eliminated from the subsequent analysis.

Concentrating on explaining the level of the dependent variable may tend to obscure other information contained in the tree which may be extremely important. The homogeneity of the final groups, especially if some of them appear after only a few splits, and are large in size, may be more interesting and important than their average on the dependent variable. Since the program produces the standard deviation as well as the mean of each group, one can examine the variance, or relative variance of each final group. If any group has a larger variance than the others, it raises the question of whether there is some other factor which affects this group, or varies more over it, but which was not included in the analysis.

The use of the tree strategy calls one's attention to the possibility that one or two variables may be sufficient for explaining the variation associated with some of the observations, whereas, additional theoretical sophistication may be required for an adequate explanation of the remainder of the sample.

Section 4.5
Overa11 Logic in Using AID (2)
The ongoing process of research in the social sciences involves both inductive and deductive reasoning (33). Theoretical orientations and conceptual schemes provide initial suggestions as to what type of data to collect. These ways of looking at the world often do not constitute a precise model, specifying exact or even probalistic relationships between clearly conceptualized and operationalized variables, nor are they often sufficiently precise to enable the deduction of specific hypotheses. But an ex-post-facto analysis of the data collection suggested by a conceptual scheme can serve as a basis for inductive reasoning, the results of which is a more precise model. Specific hypotheses can then be deduced and tried out (at least in a preliminary fashion) on the data which suggested the model from which they were deduced, and then tested on new data.


No multivariate analysis scheme can ever be a substitute for good, sound, theoretical work, but it seems clear that any one, including the AID algorithm, can be employed in both the inductive and deductive phases of research. In the inductive phase, it may be used as an aid to the formulation of a series of more precise statements about the behavior of the variables in the analysis. In the deductive phase, the tree must be consistent with the model or theoretical structure. This amounts to testing the whole model itself, rather than specific hypotheses deduced from it. The present procedure is focussed on the maximization of predictive ability. Its objective is to identify variables which discriminate between classes of observations for which predictability is good, and classes for which predictability is poor, while
providing supplementary information suggesting model refinements to take care of the latter more adequately. It is based on the conventional idea that though correlation may not be sufficient to show causation, it is necessary.

## CHAPTER V

## POSSIBLE MODIFICATIONS TO THE PROGRAM

## Section 5.1 Problems and Modifications

The work that we have done to date indicates that examining the strategy a scientist uses when working out the relationships between a few variables, formalizing it, and then extending it by means of a computer to many variables, can prove useful. The present programmed strategy is extremely limited. Certainly, additional experimentation in this type of simulation would be of value.

A number of unsolved problems with the present algorithm remain, and its usefulness could be extended by making it more sophisticated. We shall list some of the unsolved problems, propose some possible lines along which approaches to their solutions may lie, and sketch out some of the ways in which the present procedure might be extended. Then, finally, we shall take up the question of what additional modifications might be made to simulate a research analyst of somewhat greater sophistication.

The ability of the procedure to discriminate between classes of observations is based on some of the variables having important enough main effects to warrant their being used in a split. If any variable has only a very small main effect, but interacts with another variable which also has only a very small main effect, this procedure cannot discover it under certain conditions. As it stands, the class of discoverable interaction effects contains only those which involve variables, at least one of which has a detectable main effect, or which have detectable interactions with variables previously used in a split. One possible way out of this limitation would be to revise the algorithm to maximize the between-groups sums of squares one step ahead of the current step. This would involve an enumeration of all possible triads
of splits on any given parent and its two children and the sacrifice of immediate predictability in favor of better predictions further on down in the tree. We note that the tree produced by the algorithm is not necessarily that one which is better than all other possible trees for the data under consideration. It is only optimum under the sequential algorithm used. But the closer one gets to explaining all of the variation, the more likely it is that sampling variation is being explained. One buys completeness with the coin of instability.

A second problem has to do with the flexibility of the constraints that may be placed on the predictors. They are presently specified to be in one of two modes, free or monotonic. One or more modes which intermediate between the two in constraints would be desirable for variables which have a natural ordering to them, that is, either bracketed equal interval scales or ranked classes. We consider the following cases:

Case 1. The slope of the regression of the ordered class means on their identifiers does not change in sign.



Case 2. This slope changes sign once.



Case 3. The slope changes sign twice.



The dotted line represents the desired split.

The cases where the slope changes sign more than twice probably represent sampling errors or a genuine absence of correlation between that predictor and the dependent variable, and will not be discussed. It would be desirable to be able to maintain the ordering of the predictor codes, yet permit splits of the type indicated. The present monotonic mode takes care of Case 1 adequately, but is inadequate for Cases 2 and 3. Leaving the predictor in free mode may allow sampling error to exercise an undesirable amount of influence on the rank ordering of the means in such a way as to produce an erroneous split. The problem is further complicated by the fact that missing information on some predictors is usually represented as a separate class, often as a nine or a zero. If a predictor is constrained to monotonic status and Case 2 or Case 3 represents the real state of its relationship to the dependent variable, then its utility will be severely and unduly limited.

We leave aside the question of missing information and consider Case 2 and Case 3. At least two strategies are possible. For Case 2, the algorithm could be modified to split the parent group into three parts, and then either combine the two groups which are most alike (the two high's or the two low's depending on whether the $U$ is upright or inverted), or leave them as three separate groups. There are arguments for both strategies. Combining them tends to keep the group size large enough to permit a scan over all the predictors again. Keeping them separate might prove superior for theoretical reasons. If they are kept separate, subsequent frequency distributions may enable the conclusion that they have high values for different reasons.- The same strategy may be extended to Case 3.

The presence of missing information complicates things somewhat. Into which group should these observations be placed? In the free mode, they are placed together with those observations whom they are most like on the dependent variable. An alternative strategy would be to distribute them among the newly created groups on a random basis. If the algorithm were modified to accept information about which predictors contained missing information and what characters had been used to represent this, either procedure could be used to handle missing information on variables of any mode.

The present procedure requires a dependent variable which is at least assumed to be an equal interval scale, or one which is dichotomous. It would be desirable to be able to handle a dependent variable which is a series of ordered, or ranked classifications. The statistic H, presented by Kruskal and Wallis (35) might be investigated for use here.

There is a great deal that is, as yet, unknown about the present procedure, especially with reference to the rules for setting the four cut-off criteria. Some preliminary work on the distribution of the number of final groups has been done, but the mathematical relationships between group sizes, variances, skewness, the number of predictors, the number of classes and the constraint status of the predictors have yet to be worked out.

A related question has to do with the sampling stability of the trees. The tree structure itself is probably subject to more variation than the models implied by it, since there is more than one way of arriving at nearly the same set of final groups. In general, it is likely that the more complex the tree, the greater the sampling variability that can be expected. This would be in line with findings reported by Ward (36), who found that when multiple regression equations developed one sample are applied to another sample, the correlations between predicted and actual values of the dependent variable tend to decrease more when complex functions are used than when simple linear regressions are used. When data come from a sample and the model leaves out a number of the sources of variation that occur in the real world, then increasing the complexity of the interaction terms to increase explained variation can only result in greater sampling instability, since one is fitting a very precise curve to a set of points whose values are partly random. One purchases completeness with the coin of instability. The answers are to get more data and to develop a model which takes these additional sources of variation into account.

The output of the present program could be made more useful by changing the logic to cause a final sweep over all final groups regardless of their size, amount of variation, or ability to be split, and printing the statistics for each final attempted split, together with an indication of which type of final group each is.

Two additional changes could be made in the present program, making the output more usable. One would be a sumary print-out in table form, of the statistic (BSS/TSS) ${ }_{i}$. Another would be better identification of final groups for which one or more of the predictors is a constant or is heavily clustered.

Another possibility would be the incorporation of a procedure for automatic scanning to detect the trunk-twig structure that indicates skemmess. Or measures of skewness and kurtosis could be computed in advance of each attempted split and a decision made as to whether to attempt to locate a discriminator that would split off the observations in the tail, rather than explaining maximum variation. Examining the shape of the distribution of the dependent variable before each spiit might also provide the basis for a decision as to whether to split a group into two or three parts.

Still another addition to the algorithm which might be useful is the automatic pooling, or combining of final groups with similar mean values. This should probably be applied only to small, and to unexplainable final groups, and would involve the sacrifice of some explained variation, because the means of these groups would not be identical. But combining some of these groups might well make possible additional splits that would more than offset the losses. The subsequent groups might be very difficult to describe or explain, however.

Since the logic of elaboration and specification is heavily dependent on intercorrelations between predictors, it would be desirable to incorporate into the program the instructions necessary to compute an intercorrelation matrix of predictors associated with each split. This would enable the analyst to follow the patterns of change in the intercorrelations from split to split.

The reader should note that in some of the above suggested lines of modification, we are proposing to incorporate into the program some of the decisions that the analyst himself is, at present, making. This is particularly true in the detection of the trunk twig structure that indicates skewness. We ask the question, "What information does the analyst use to make a decision, what are his alternative lines of action, and on what basis does he choose one line of action over
another?" If the information is already in the computer and if the basis for decision has a clear criterion and can be formalized, then it can be programmed.

One last line of promising development is suggested. Westervelt (11) has shown that artificial intelligence may be applied successfully to a sequential algorithm aimed at maximizing predictive power. .He incorporated a simple learning procedure into the now well-known stepregression technique. Information about how to solve the problem is built up through experience with attempts to solve it. Thus, a further extension of the AID algorithm might well involve a series of trial trees which were not restricted to the best split at each stage, but chose on a random basis among those predictors which were almost equally good, and which produced information about what works and under what circumstances. By repeated iterations, modifying the probabilities with which each variable is used in each split on the combined basis of its effects in that split and the efficiency of subsequent splits, it may be possible to produce a tree which explains a great deal more of the variation in the dependent variable than that presently obtained.

Section $5.2 \quad$ Strategy and Computers
The foregoing presentation has been based on the presently predominant method of using a large-scale digital computer, batchprocessing. By this, we mean a machine-use mode in which problems are submitted to the computer in a stream--one after the other. In this mode, a problem is completely processed before another is started, and it is desirable for the analyst to get as much information out of a "job" as he can use.

This is not the only mode of machine organization. As computers increase in size and speed, the possibilities of the simultaneous processing of many problems grow highly probable. Indeed, at least one computer installation (37) is now experimenting with a remote console mode of operation. The analyst can then be brought into direct and immediate communication with the computer, re-acquiring not only the ability to intervene directly in the computing process (an ability severely lacking in the batch-processing mode of operation), but being able to do so with a great deal more power than he had when looking at banks of counters on a sorter. Moreover, programming techniques for translating problem-oriented languages similar to English into machine instructions are now developed to the point where direct on-line communication with a large scale computer operating in multi-processing mode is quite feasible.

This implies that far from being cloud-nine thinking, the distinct possibility of the analyst sitting at his desk with a console typewriter and requesting information from the computer is a realistic possibility. The following examples of possible requests might be typical of such a situation:

1. DISPLAY THE INTERCORRELATIONS BETWEEN X(1) AND X(2) IN GROUP 7.
2. DISPLAY AN UNSORTED, TENTATIVE, SPLIT OF GROUP 6 ON X(5).
3. CONTINUE AN AUTOMATIC ANALYSIS ON GROUPS 6, 9, AND 13.
4. DISPLAY THE (BSS/TSS) ${ }_{i}$ TABLE FOR GROUPS 6, 7, AND 9.

This would allow the analyst to insert his hunches into the computing process.

## CHAPTER VI

## SUMMARY AND CONCLUSIONS

Summary and Conclusions
Our starting point has been a consideration of some of the problems inherent in the application of multivariate statistical techniques to survey data (3). Most of the problems of analyzing this type of data have been reasonably well handled, except those revolving around the existence of interaction effects. A number of multivariate techni ques are now in use, but this increased efficiency has been achieved primarily by imposing linearity and addịtivity assumptions. Since many useful concepts are classifications, their introduction into conventional multivariate procedures are difficult. Moreover, these procedures tend to obscure rather than illuminate errors in the measurement process. The fact that almost all survey samples are stratified and clustered leads to severe problems in the proper applications of statistical tests of significance. The intercorrelations between explanatory factors and interactions between them, make difficult the construction of precise theoretical models reflecting chains of causation, especially where the number of explanatory factors is large.

The procedure presented here represents an attempt to attack some of these problems by asking different kinds of statistical questions of the data than are implied by the choice of multiple regression techniques.

It is capable of handing a large number of predictors, will handle variables which are only nominal scales (i.e., mere classifications), and appears to be somewhat sensitive to measurement error. Linearity of relationships is not assumed. The problem of whether or not something could reasonably have occurred by chance will be with us as long as sampling techniques are used; but we suggest that the proper
focus of the analysis should be on explanatory power, or importance, not significance. It is this focus which underlies what has been presented.

In the inductive phases of science the problem is to develop a model that fits the observed patterns of relationships between variables maximally. It is unlikely that a model which does not predict well for the sample upon which it is based will prove useful for very long without extensive modifications. Multivariate statistical methods are one of the tools used to develop such models. It is our hope that we have added a useful one to the tool-kit.

## APPENDIX A

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## AID (Model 2) FORMULAS

FOR TOTAL:
$Y_{\alpha}=$ Value of dependent variable for the $\alpha$ th observation in the data
$\mathrm{N}=$ Total number of observations in the data
$w_{\alpha}=\quad \begin{aligned} & \text { Weight value attached to the } \alpha \text { th observation in the } \\ & \text { data }\end{aligned}$
$w_{\alpha}=1$ if the data is unweighted
$\mathrm{W}=\sum_{\alpha=1}^{N}{ }^{\mathrm{w}} \alpha=$ sum of weights*
$\Sigma Y=\sum_{\alpha=1}^{N} W_{\alpha} Y=$ sum of $Y$
$\Sigma Y^{2}=\sum_{\alpha=1}^{N} w_{\alpha} Y_{\alpha}^{2}=$ sum of $Y$-squared
$\bar{Y}=\frac{\Sigma Y}{W}=$ mean
$\sigma=\sqrt{\frac{1}{W}\left(\Sigma Y^{2}-\frac{(\Sigma Y)^{2}}{W}\right)}=$ standard deviation*
$T S S_{T}=\Sigma Y^{2}-\frac{(\Sigma Y)^{2}}{W}=$ total sum of squares
${ }_{B S S}=\sum_{i=1}^{K} \frac{\left(\Sigma Y_{i}\right)^{2}}{W_{i}}-\frac{(\Sigma Y)^{2}}{W}=\begin{aligned} & \text { between-group sum of squares where } \\ & \text { the summation }(i=1,2, \ldots, k-1, k)\end{aligned}$ is over the final unsplit groups
$\mathrm{WSS}_{\mathbf{T}}=\mathrm{TSS}_{\mathbf{T}}-\mathrm{BSS}_{\mathbf{T}}=$ within-group sum of squares

[^2]Gross Beta Coefficient $B_{\bar{x}}^{2}$; the proportion of variance which could be explained by predictor $x$ alone in a one-way analysis of variance over its $c_{x}$ classes

$$
\mathrm{B}_{\mathrm{x}}^{2}=\frac{\mathrm{BSS}_{\mathrm{x}}}{\mathrm{TSS}} \mathrm{~T}
$$

where $\mathrm{TSS}_{\mathrm{T}}$ is defined as above and

$$
B S S_{x}=\sum_{k=1}^{C_{x}} \frac{\left(\Sigma Y_{k}\right)^{2}}{W_{k}}-\frac{(\Sigma Y)^{2}}{W}
$$

and $C_{x}$ is the number of classes defined by predictor $x$.

Partial Beta Coefficient $\beta_{x}^{2}$; the proportion of variance explained by predictor $\mathbf{x}$ in the tree.

$$
\beta_{x}^{2}=\frac{\sum_{i T S S_{i x}}-\sum_{j} \mathrm{TSS}_{j x}}{\operatorname{TSS}_{\mathrm{T}}}
$$

where $i$ is over all parent groups split by predictor $x$, and $j$ is over all new groups formed by splitting a parent group on predictor x . An equivalent computational formula using program output is:

$$
\beta_{x}^{2}=\sum_{i} \frac{T_{S S}{ }_{i x}}{T S S_{T}}-\sum_{j} \frac{\operatorname{TSS}_{j x}}{T S S_{T}}
$$

The total proportion of variance explained by the tree is:

$$
\mathrm{R}^{2}=\sum_{\mathrm{x}=1}^{\mathrm{NP}} \beta_{\mathrm{x}}^{2}=\frac{\mathrm{BSS}_{\mathrm{T}}}{\mathrm{TSS}_{\mathrm{T}}}
$$

where $N P$ is the number of predictors used in the analysis.

The reduction in unexplained variation from any one split is

$$
\mathrm{D}=\frac{\mathrm{TSS}_{i}}{\mathrm{TSS}_{\mathrm{T}}}-\left(\left[\frac{\mathrm{TSS}}{\mathrm{TSS}_{\mathrm{T}}}\right]+\left[\frac{\mathrm{TSS}_{\mathrm{k}}}{\mathrm{TSS}_{\mathrm{T}}}\right]\right)
$$

where $i$ is the identifier of the group being split and $j$ and $k$ are the identifiers of the resultant groups.

Formulas for the isth Group

$$
\begin{aligned}
& N_{i}=\text { number of observations in the } i^{\prime} \text { th group } \\
& W_{i}=\sum_{\alpha=1}^{N_{i}} W_{\alpha}=\text { sum of weights in the } i^{1} \text { th group } \\
& \Sigma Y_{i}=\sum_{\alpha=1}^{N_{i}} w_{\alpha} Y_{\alpha}=\text { sum of } Y \text { in the } i \text { 'th group } \\
& \Sigma Y_{i}^{2}=\sum_{\alpha=1}^{N_{i}} W_{\alpha} Y_{\alpha}^{2}=\text { sum of } Y \text {-squared in the } i^{\prime} \text { th group } \\
& \bar{Y}_{i}=\text { mean of the } i^{\prime} \text { th group }
\end{aligned}
$$

$$
\begin{aligned}
& \sigma_{i}=\sqrt{\frac{\operatorname{TSS}_{i}}{W_{i}}} \\
& D_{i}=\bar{Y}_{i}-\bar{Y}=\text { deviation of the mean of the } i^{\prime} \text { th group } \\
& \text { from the grand mean } \\
& \frac{T S S_{i}}{T S S_{T}}=\begin{array}{l}
\text { proportion of the original total sum of squares still } \\
\text { left in the } i
\end{array} \\
& \frac{W_{i}}{W} \times 100=\begin{array}{l}
\text { per cent of total }=\text { (weighted) } \\
\text { observations in the } i \text { 'th group }
\end{array} \text { proportion of the } \\
& \frac{\left(\sum_{\alpha_{i=1}}^{N_{i}} W_{\alpha} Y^{2}\right.}{W_{i}}=\text { weighted mean square for the } i^{\prime} \text { th group } \\
& \text { PA }=\text { PCT (an input parameter) times } \mathrm{TSS}_{T} \text {. If the isth group } \\
& \text { is to become a candidate for splitting, then } P A \leqq T S S_{i} \text {. }
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{PB}= \text { PCT2 (an input parameter) times } \mathrm{TSS}_{\mathrm{T}} \text {. In order for an } \\
& \text { attempted split on group } \mathrm{i} \text { to be allowed the require- } \\
& \text { ment } \mathrm{PB} \leqq \mathrm{BSS}_{\mathrm{mpi}} \text { must be met. }
\end{aligned}
$$

$$
B S S_{m p i}=\frac{\left(\sum_{j=0}^{m} Y_{j}\right)^{2}}{\Sigma W_{j}}+\frac{\left(\sum_{j=m+1}^{c} Y_{j}\right)^{2}}{\Sigma W_{j}}-\frac{\left(\sum_{j=0}^{c} Y_{j}\right)^{2}}{\Sigma W_{j}}
$$

where $m$ is the split between the $m^{\prime} t h$ and $m+l s t$ classes of predictor p over group i. $C$ is the maximum value attained by predictor $p$.

BSS $_{\text {mpi }}$ is maximized over all classes of all predictors over group i. -

The split of group $i$ occurs after selection of the maximum $B S S_{m p i}$ and occurs only if the criterion $\mathrm{PB} \leqq \mathrm{BSS}_{\mathrm{mpi}}$ is met. There are $\mathrm{C}-1$ elements in the BSS column produced by the partition scan output. These are the $B S S_{m p i}$. The $C^{\prime}$ th element is $T S S_{i}$ for the group being split. Ratio of variation in group i explained by unsuccessful predictor $r$ in attempted partitioning of group $i, \operatorname{BSS} / \mathrm{TSS}=\mathrm{BSS}_{\mathrm{mpr}} / \mathrm{TSS}_{\mathrm{i}} \cdot$

## APPENDIX C

A Note on<br>Partitioning for Maximum Between<br>Sum of Squares<br>11/10/62<br>by W. A. Ericson

## 1. The Problem

This note presents some results, both positive and negative, concerned with analysis of the following problem:

One is given $k>2$ sets of observations, where

$$
\bar{x}_{i}, \quad i=1,2, \ldots, k
$$

is the mean of the observations within the i'th set and

$$
N_{i}, \quad i=\dot{1}, 2, \ldots, k
$$

is the number of observations in that set. The problem is to partition these $k$ sets of observations into two nonempty classes such that the "between class sum of squares" is maximized. In other words, to find $I$, a set of any $m(1 \leq m<k)$ of the $k$ indices $i=1,2, \ldots, k$, such that

$$
\begin{equation*}
N_{I}\left(\bar{x}_{I}-\bar{x}\right)^{2}+N_{\bar{I}}\left(x_{\bar{I}}-\bar{x}\right)^{2} \tag{1}
\end{equation*}
$$

is maximized, where

$$
\begin{aligned}
& N_{I}=\sum_{i \in I} N_{i}, \quad N_{\bar{I}}=\sum_{i \notin I} N_{i}, \\
& \bar{x}_{I}=\frac{1}{N_{I}} \sum_{i \in I} N_{i} \bar{x}_{i}, \quad \bar{x}_{\bar{I}}=\frac{1}{N_{\bar{I}}} \sum_{i \notin I} N_{i} \bar{x}_{i},
\end{aligned}
$$

and $\bar{x}$ is the overall mean, i.e.,

$$
\bar{x}=\frac{N_{I} \bar{x}_{I}+N_{\bar{I}_{\bar{I}}} \bar{x}_{\bar{I}}}{N_{I}+N_{\bar{I}}} .
$$

## 2. Previous Results

No literature search having been made, it is not known whether this problem has been researched by other investigators. This remains a point for further study.
3. Restatement and Assumptions

It is well-known that the problem outlined above is basically unchanged by the addition of the same arbitrary constant to each $\bar{x}_{i}$. It may thus be assumed without loss of generality that

$$
\begin{equation*}
\bar{x}_{1} \geq \bar{x}_{2} \geq \ldots \geq \bar{x}_{k}>0 \tag{2}
\end{equation*}
$$

Furthermore, it is easily seen that maximizing (1) by choice of I is equivalent to maximizing

$$
\begin{equation*}
f(I) \equiv \frac{\left(N_{I} \bar{x}_{I}\right)^{2}}{N_{I}}+\frac{\left(N_{\bar{I}} \bar{x}_{\bar{I}}\right)^{2}}{N_{\bar{I}}} \tag{3}
\end{equation*}
$$

## 4. A Negative Result

The following algorithm was suggested for finding $I$ and its complement, $\overline{\mathrm{I}}$, which maximizes (3):
a) Compute $f(I)$ for $I$ taken, in turn to be $\{1\},\{2\}, \ldots,\{k\}$.
b) Pick the maximum $f(I)$ over these I's. Suppose, egg., $I=\{a\}$ maximizes $f(I)$ over the I's considered in (a).
c) Compute $f(I)$ for $I$ taken in turn to be $\{a, 1\}, \ldots,\{a, a-1\},\{a, a+1\}$, $\ldots\{a, k\}$.
d) Choose that $I$, among those considered in (c) which maximizes $f(I)$, say $I=\{a, b\}$. If $f(\{a\})>f(\{a, b\}), \quad$ stop and assert $I=\{a\}$
yields maximum value of (3), otherwise continue the process, looking next at $f(I)$ for $I$ 's of the form $\{a, b, i\}, i \neq a, i \neq b$, repeating steps (c) and (d) above.

This procedure does not lead invariably to the optimum or maximizing partition, I. That this is so is demonstrated by the following counterexample:

Suppose $k=5$ and the data are as shown below:

| $i:$ | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\bar{x}_{i}:$ | 3.1 | 3.0 | 2.0 | 2.0 | 1.0 |
| $N_{i}:$ | 1 | 2 | 3 | 1 | 3 |

It is easily verified that

| $I$ | $\overline{\mathrm{I}}$ | $f(\mathrm{I})$ |
| :---: | :---: | :---: |
| $\{1\}$ | $\{2,3,4,5\}$ | 41.72111 |
| $\{2\}$ | $\{1,3,4,5\}$ | 42.85125 |
| $\{3\}$ | $\{1,2,4,5\}$ | 40.40142 |
| $\{4\}$ | $\{1,2,3,5\}$ | 39.31764 |
| $\{5\}$ | $\{1,2,3,4\}$ | 44.77285 |

Following the suggested algorithm we next look at $I=(5, i)$, $i=1,2,3,4$, and obtain the following:

| $I$ | $\overline{\mathrm{I}}$ | $\mathrm{f}(\mathrm{I})$ |
| :---: | :---: | :---: |
| $\{5,1\}$ | $\{2,3,4\}$ | 41.96916 |
| $\{5,2\}$ | $\{1,3,4\}$ | 40.84200 |
| $\{5,3\}$ | $\{1,2,4\}$ | 44.30250 |
| $\{5,4\}$ | $\{1,2,3\}$ | 44.25166 |

Each of these values of $f(I)$ being less that $f(\{5\})$, we conclude, as per the suggested algorithm, that $I=\{5\}$ maximizes (3). This is not true since it is easily shown that

$$
f(\{1,2\})=44.88904>f(\{5\})=44.77285
$$

5. The Basic Result

It will be proved in this section that (3) is maximized over all possible $I^{\prime} s$ by $I^{*}$ where $I^{*}$ is that set $I_{m} \equiv\{1,2, \ldots, m\}, 1 \leq m<k$ for which $f(I *) \geq f\left(I_{m}\right)$ for all m. Thus to find the maximizing partition one need only compute $f(I)$ for the $k-1$ sets $I_{m}$ and choose the maximum. Furthermore, I*, obtained in this fashion, maximizes (3) over any partition of the $N=\sum_{1}^{k} N_{i}$ individual observations into two sets (assuming each individual observation within any set equals the set mean $\bar{x}_{i}$ say).

The present proof of these assertions, while straightforward, involves considerable tedious algebra. Further study may yield more succinct and more tidy demonstrations. The present proof is given in two parts. We first state and prove the theoretical results, in some degree of generality and then make the necessary identifications to the problem stated in 81 by which the assertions stated above become estab1ished.

We adopt the following notation: let

$$
\begin{equation*}
a_{1} \geq a_{2} \geq a_{3} \geq \ldots \geq a_{N} \tag{4}
\end{equation*}
$$

be any nonincreasing sequence of real positive numbers. Let $P_{m}$ and $P_{n}$ be any partition of the $N a_{i}$ 's, i.e., $P_{m}$ is any set of $m$ of the $a_{i}$ 's and $P_{n}$ is the set of the remaining $n=N-m$ $a_{i}$ 's. Further, let $H_{m}, L_{m}$ and $M$ be respectively the set of the largest $m a_{i}{ }^{\prime} s$, the set of the smallest $m a_{i}{ }^{\prime} s$, and the $n-m$ middle $a_{i}{ }^{\prime} s$. (It is assumed that $n \geq m$, hence $M$ is null if $\mathrm{n}=\mathrm{m}$, otherwise not.) Thus

$$
\begin{aligned}
H_{m} & =\left\{a_{1}, \ldots, a_{m}\right\} \\
L_{m} & =\left\{a_{N-m+1}, \ldots, a_{N}\right\} \\
M & =\left\{a_{M+1}, \cdots, a_{N-m}\right\}
\end{aligned}
$$

The first result may then be stated as
Theorem A: At least one of the following is true:
a) $\quad \frac{\left(\Sigma\left(H_{m}\right)\right)^{2}}{m}+\frac{\left(\Sigma(M)+\Sigma\left(L_{m}\right)\right)^{2}}{n} \geq \frac{\left(\Sigma\left(P_{m}\right)\right)^{2}}{m}+\frac{\left(\Sigma\left(P_{n}\right)\right)^{2}}{n}$
b) $\frac{\left(\Sigma\left(L_{m}\right)\right)^{2}}{m}+\frac{\left(\Sigma(M)+\Sigma\left(H_{m}\right)\right)^{2}}{n} \geq \frac{\left(\Sigma\left(P_{m}\right)\right)^{2}}{m}+\frac{\left(\Sigma\left(P_{n}\right)\right)^{2}}{n}$,
where $\Sigma\left(H_{m}\right) \equiv \sum_{i} \sum_{\mathrm{E}_{\mathrm{m}}} \mathrm{a}_{\mathrm{i}}$, etc.
$\nabla$ Proof: The theorem is obviously true if either $\Sigma\left(L_{m}\right)=\Sigma\left(P_{m}\right)$ or $\Sigma\left(H_{m}\right)=\Sigma\left(P_{m}\right)$. We then consider the other cases, i.e., $\Sigma\left(H_{m}\right)>\Sigma\left(P_{m}\right)>\Sigma\left(L_{m}\right)$, and show that if (a) fails then (b) holds. Straightforward algebra* shows that if (a) is false, then

$$
\begin{equation*}
\left[m \Sigma\left(P_{n}\right)+m\left(\Sigma\left(L_{m}\right)+\Sigma(M)\right)-n\left(\Sigma\left(H_{m}\right)+\Sigma\left(P_{m}\right)\right)\right]>0 \tag{5}
\end{equation*}
$$

Similarly, (b) is true if

$$
\begin{equation*}
\left[m \Sigma\left(P_{n}\right)+m\left(\Sigma(M)+\Sigma\left(H_{m}\right)\right)-n\left(\Sigma\left(L_{m}\right)+\Sigma\left(P_{m}\right)\right)\right] \geq 0 \tag{6}
\end{equation*}
$$

That (5) implies (6) is obvious, since the left side of (6) is greater than or equal to the left side of (5).

[^3]The second main result is given by the following:

Theorem B: Suppose

$$
\begin{aligned}
a_{1} & \geq \ldots \geq a_{m}>a_{m+1}=\ldots=a_{m+n} \\
& =a_{m+n+1}=\ldots=a_{m+n+l}>a_{m+n+l+1} \geq \ldots \geq a_{m+n+l+r}
\end{aligned}
$$

where $m+n+\ell+r=N, m \geq 0, n>0, \quad \ell>0, r \geq 0$, and $m+r \geq 1$. Then at least one of the following statements is defined and true:
d) $\frac{1}{m}\left(\Sigma_{m}\right)^{2}+\frac{1}{n+\ell+r}\left((n+\ell) a+\Sigma_{r}\right)^{2} \geq \frac{1}{m+n}\left(\Sigma_{m}+n a\right)^{2}+\frac{1}{\ell+r}\left(\ell a+\Sigma_{r}\right)^{2}$
or
d) $\frac{1}{m+n+\chi}\left(\Sigma_{m}+(n+\ell) a\right)^{2}+\frac{1}{r}\left(\Sigma_{r}\right)^{2} \geq \frac{1}{m+n}\left(\Sigma_{m}+n a\right)^{2}+\frac{1}{\ell+r}\left(\ell_{a}+\Sigma_{r}\right)^{2}$, where $a \equiv a_{i}, \quad i=m+1, \ldots, m+n+\ell, \quad \Sigma_{m} \equiv \sum_{i=1}^{m} a_{i}$, $\Sigma_{r} \equiv \sum_{i=1}^{r} a_{m+n+f+i}$.

Proof: If $m=0$, it is immediately verifiable that (d) is true. Likewise, if $r=0$, then ( $c$ ) is true. Suppose then that $m, n$, $r$, and $\ell$ are all positive. Straightforward algebra shows that
(c) is equivalent to
c) $\quad A \equiv\left(\Sigma_{m}\right)^{2}-2 m a \Sigma_{m} \geq \frac{m(m+n)}{(n+\ell+r)(\hat{\ell}+r)}\left(\Sigma_{r}\right)^{2}-\frac{2 m r(m+n)}{(n+\ell+r)(\ell+r)} a \Sigma_{r}$ $+\frac{m[(m+n) \hat{\ell}-(\ell+r) n]^{2}-m\left[(m+n) \ell^{2}+(\ell+r) n^{2}\right]}{n(n+\ell+x)(\ell+r)} a^{2} \equiv B$
and (d) is equivalent to:
d') $A \equiv\left(\Sigma_{m}\right)^{2}-2 m a \leq \frac{(m+n+l)(m+n)}{r(l+r)}\left(\Sigma_{r}\right)^{2}-\frac{2(m+n+\ell)(m+n)}{(l+\dot{r})} a \Sigma_{r}$

$$
-\frac{\left\{[(m+n) \ell-(\ell+r) n]^{2}-r\left[(m+n) l^{2}+(\ell+r) n^{2}\right]\right\}}{\ell(l+r)} a^{2} \equiv c .
$$

To show that either (c) or (d) is true (or both) it suffices then to show that if ( $c^{\prime}$ ) is false then ( $d^{\prime}$ ) must be true. This is clearly established if the right side of the inequality in ( $c^{\prime}$ ) is less than or equal to the right side of the inequality in ( $\mathrm{d}^{\prime}$ ), i.e., if $C-B \geq 0$. But some simple but tedious algebra shows that

$$
c-B=\frac{(m+n)[(n+\ell+r)(m+n+\ell)-m r]}{r(n+\ell+r)(\ell+r)}\left[\sum_{r}-r a\right]^{2},
$$

which is obviously nonnegative. A
To use these results for the problem stated in $\$ 1$ above and to establish the assertions at the beginning of the present section one need only identify the following nonincreasing sequence with those sequences of $a_{i}{ }^{\prime} s$ referred to above:


Then it is clear that Theorem A establishes the fact that for any partition of these $N=\sum_{l}^{k} N_{i} \bar{x}_{i}$ 's into two sets of $m$ and $\mathrm{n}=\mathrm{N}-\mathrm{M}$ elements respectively will yield a value of "between sum of squares," (3), no larger than that for either the partition consisting of the $m$ largest $\bar{x}_{i}{ }^{\prime} s$ and the $N-m$ remaining or the $m$ smallest $X_{i}$ 's and the $N-m$ remaining. This result clearly includes the case where for every $i=1, \ldots, k$ all $N_{i} \bar{X}_{i}$ 's are put in the same one of the two sets forming the partition, i.e., the case where the partition is of the $k$ sets of means rather than of the $N$ individual means.

Theorem $B$ then closes the remaining loophole, viz., it may be that some partition, $I$, $\bar{I}$, of the $k$ sets of means into $N_{I}=\sum_{i \in I} N_{i}$ and $N_{\bar{I}}=N-N_{I}$ observations, respectively, has a sum of squares, (3), which is no larger than that for the partition consisting, say, of the largest $N_{I}$ individual $\bar{x}_{i}$ 's and the $N_{\bar{I}}$ remaining $\bar{x}_{i}$ 's. However, this latter partition may very easily split one set of $N_{i}$ identical $\bar{x}_{i}$ 's. Theorem $B$ then says that for any partition of the $N$ individual $\bar{x}_{i}$ 's into the $m$ largest
and $N-m$ remaining and where the partitioning point occurs within one of the $k$ sets of observations then there is another partition into largest and smallest $\bar{x}_{i}$ 's where the partitioning point occurs between two of the $k$ sets of $\bar{x}_{i} ' s$ and which has a between sum of squares no smaller than the original partition.

Theorems A and B then together demonstrate that to find the partition which maximizes (3) one need only look at the $k-1$ partitions, $I_{m}$, where $I_{m}=\{1,2, \ldots, m\}, 1 \leq m<k$, and choose that one yielding the largest value of (3).

## 6. A Final Negative Result

It was further conjectured that perhaps (3), $f\left(\left\{I_{m}\right\}\right)$, $m=1,2, \ldots, k-1$, treated as a function of $m$ was well-behaved in the sense of say concavity and that, e.g., if $f\left(\left\{I_{1}\right\}\right)>f\left(\left\{I_{2}\right\}\right)$ then one might be able to stop and assert $I^{*}=I_{1}$, and thus not look at all $k-1 I_{m}$ s. This is not the case, however, as witnessed by the following counter example:

| i | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\overline{\mathrm{x}}_{\mathrm{i}}$ | 3.000 | 2.01000 | 2.0010 | 2.0001 | 1.0000 |
| $\mathrm{~N}_{\mathrm{i}}$ | 1 | 1 | 1 | 1 | 2, |

here one finds the following values for $f\left(\left\{I_{m}\right\}\right), m=1,2,3,4$ :

| $\underline{I_{m}}$ | $\underline{f\left(\left\{I_{m}\right\}\right)}$ |
| :---: | :---: |
| \{1\} | 21.84 |
| $\{1,2\}$ | 21.55 |
| $\{1,2,3\}$ | 21.72 |
| $\{1,2,3,4\}$ | 22.30 |

## 7. Conclusions

The above results indicate that to find the partition which maximizes the between sum of squares, (3), one need only compute (3) for the $k-1$ partitions consisting of the first set of size $N_{1}$ and all the rest, the first two sets of size $N_{1}+N_{2}$ and all the remaining, etc., and choosing that one which maximizes (3). Further the partition found in this manner maximizes (3) over all partitions of the $N=\sum_{1}^{k} N_{i}$ individual observations (assuming each observations within any one of the $k$ sets equals the mean of that set). Finally it does not seem possible to improve on this technique, in the sense of reducing the computational burden.

## APPENDIX D

## AID (2) ALGORITHM

## Preliminary Read in. Steps 1 and 2.

1. Read in all parameters and all input observations, including all predictors and the dependent variable Y. Screen out observations where Y is missing data or it is not desired to use this observation. Save all observations on tape if necessary.
2. To start, identify all observations used in the analysis as belonging to group number one. Group number one is the current candidate group. Go to Step 6.

## Test for Termination of the Procedure. Step 3.

3. Determine whether or not the current number of unsplit groups is about to exceed the maximum permissible number; if so, go to Step 22, as the problem cannot proceed further.

> Determine Which Group Should Be Selected for Attempted Partitioning. Steps 4-6.
4. Considering all groups constructed so far, find one of them such that
a. the total sum of squares $\left(\mathrm{TSS}_{\mathrm{i}}\right)$ of that group is greater than or equal to $R$ per cent of the total sum of squares for the input observations (TSS ${ }_{t}$ );
b. the number of observations in the group is not smaller than MSIZE;
c. the group has not already been split up into two other groups;
d. there has been no previous failure to split up the group;
e. the total sum of squares of that group is not smaller than the sum of squares for any other group that meets the above four criteria.
5. If there is no such group, go to Step 23; the problem is complete.
6. The group selected is the current candidate group, which will be the subject of an attempted split. Identify it with its group number (i) and print out $N_{i}, \Sigma Y_{i}, \Sigma Y_{i}^{2}, \bar{Y}_{i}$, and $T S S_{i}$.

## Partition Scan Over All Predictors. Steps 7-19.

7. Set $j=1$ and go to Step 9 .
8. Increment $j$ by 1 . If $j$ is larger than the number of predictors being used in the analysis, the partition scan is complete; go to Step 20.
9. Compute $N_{i j c}, \Sigma Y_{i j c}, \Sigma Y_{i j c}^{2}, \bar{Y}_{i j c}$ for each class c of predictor $j$ over group i.
10. Determine whether or not there exist two or more classes $c$, such that $N_{i j c} \neq 0$. If not, predictor $j$ is a constant over group $i ;$ print an appropriate comment and go to step 8.
11. If predictor j has been defined as monotonic, skip Step 12 , do not sort the Step 9 statistics, go to Step 13 instead.
12. Sort the statistics produced in Step 9, together with the class identifiers for predictor $j$, into descending sequence using $\bar{Y}_{i j}$ as a key.

Partition Scan Over the $c$ Classes of Predictor $j$. Steps 13-17.
13. Set $p=1$ and go to Step 15.
14. Increase $p$ by 1 . If $p$ is larger than ( $c_{j}-1$ ), where $c_{j}$ is the number of classes in the $j^{\prime}$ th predictor, then print the statistics for class $c_{j}$ and go to Step 18 as all possible feasible splits have been examined.
15. If $\Sigma N_{k}=N_{1}=0$ for $k=1, \ldots p$, or if $\left(N_{i}-N_{1}\right)=N_{2}=0$, go to Step 14 as this split cannot be made because of empty classes in this group for predictor $j$. Otherwise, compute BSS $_{p}$, the betweengroups sum of squares for the attempted binary split of group $i$ on predictor $j$ between the sorted classes ( $1, \ldots, p$ ) and the adjacent sorted classes ( $p+1, \ldots, c$ ). Print the statistics for class p.
16. If this BSS $_{p}$ is not larger than any BSS $_{p}$ previously computed for this predictor over this group, go to ${ }^{P}$ Step 14.
17. This is the largest BSS $_{p}$ encountered so far for this predictor. Remember $\mathrm{BSS}_{\mathrm{p}}$ and the partition number p ; print them and go to Step 14.

## Determination of Best Predictor. Steps 18-19.

*18. Was the maximum BSS $_{p}$ for predictor $j$ larger than the largest BSS $_{p}$ obtained from any of the other predictors previously tested over ${ }^{p}$ group i? If not, go to Step 8.
19. This is the best $\mathrm{BSS}_{\mathrm{p}}$ produced by any of the predictors tested so far over group i. Remember this partition and this predictor and then go to Step 8.

## Is the Best Predictor Worth Using? Steps 20-21.

*20. Was the maximum BSS retained after the scan of all predictors over group $i$ equal to at least $Q$ per cent of the total sum of squares? If not, mark group $i$ as having failed in a split attempt and then go to Step 4.
21. Group i is to be split into two new groups and destroyed. Using the class identifiers and the partition rule remembered from Step 19, split the observations in group i into two parts. Identify the two new groups as having been created. Identify group i as having been split. Print the statistics from the successful partition attempt. Increase the total number of groups created so far by the quantity 2. Increase the current number of unsplit groups by one. Then go to Step 3.

Termination of the Algorithm. Steps 22-26.
22. The maximum number of permissible unsplit groups has been reached. Print an appropriate comment and go to Step 24.
23. There are no more groups eligible for further splitting. Print an appropriate comment and to go Step 24.
24. Print out a summary record of all groups created in the process of splitting, including the group number, its parent group, the values of the predictor class identifiers that were used in the partition which constructed the group, the predictor number used in this partition, an indication of whether or not this present group was ever split, and $\mathrm{N}_{\mathrm{i}}, \Sigma \mathrm{Y}_{\mathrm{i}}, \Sigma \mathrm{Y}_{\mathrm{i}}^{2}$, and $\mathrm{TS} \mathrm{S}_{\mathrm{i}}$.
25. Determine whether punched or tape residuals are desired. If so, go to Step 26, otherwise go to Step 1.
26. Compute predicted values of $Y$ and residuals and, by option, punch them and/or write them on tape with the data. Then go to Step 1.

[^4]
## Formulas

$$
\begin{array}{rlr}
\bar{Y} & =\Sigma Y / N \\
\text { TSS } & =\Sigma Y^{2}-\frac{(\Sigma Y)^{2}}{N} & Y_{\alpha}=\bar{Y}_{i} \\
B S S & =\frac{\left(\Sigma Y_{1}\right)^{2}}{N_{1}}+\frac{\left(\Sigma Y_{2}\right)^{2}}{N_{2}}-\frac{(\Sigma Y)^{2}}{N} & R_{\alpha}=\vec{Y}_{\alpha}-Y_{\alpha}
\end{array}
$$

$$
\mathrm{WSS}=\mathrm{TSS}-\mathrm{BSS}
$$

## AID (2) Algorithm: Summary

1. Considering the currently unsplit sample subgroups having at least 25 observations in them, select that sample subgroup which has the largest total sum of squares, such that $\mathrm{TSS}_{\mathrm{i}} \geq \mathrm{R}\left(\mathrm{TSS}_{\mathrm{T}}\right)$

$$
T S S_{i}=\Sigma Y_{i}^{2}-\frac{(\Sigma Y)^{2}}{N_{i}}
$$

The total sample is considered the first (and indeed, only) such group at the start.
2. Find the division of the classes of any single characteristic such that the partition $p$ of this group into two subgroups on this basis provides the largest reduction in the unexplained sum of squares. Choose a division so as to maximize

$$
\left(\mathrm{N}_{1} \overline{\mathrm{Y}}_{1}^{2}+\mathrm{N}_{2} \overline{\mathrm{Y}}_{2}^{2}\right)
$$

with the restrictions that (1) the classes are ordered in descending sequence using their means as a key and (2) observations belonging to classes which are not contiguous are not placed together in one of the new groups to be formed. (3) The sorting of classes may be suppressed by option.
3. For a partition $p$ on variable $k$ over group $i$ to take place after the completion of (2), it is required that:

$$
\left(N_{1} \bar{Y}_{1}^{2}+N_{2} \bar{Y}_{2}^{2}\right)-N_{i} \bar{Y}_{i}^{2} \geq Q\left(\Sigma Y_{T}^{2}-N \bar{Y}_{T}^{2}\right)
$$

Otherwise group i is not capable of being split. No variable is "useful" over this group. The next most promising group $\left(\mathrm{TSS}_{\mathrm{i}}=\max \right)$ is selected.
4. If there are no more groups such that $T S S_{i} \geq R\left(T S S_{T}\right)$, or if for the groups that meet this criterion there is no "useful" variable, or if the number of unsplit groups exceeds a specified number, the process terminates.
flow Charts
AID (2)
PROGRAM SEGMENT 1 (EDITOR)


FLOW CHARTS
AID (2)
PROGRAM SEGMENT 1 (EDITOR)






FLOW CHARTS
AID (2)
PROGRAM SEGMENT 3 (OUTPUT)


APPENDIX F<br>COMPUTER PROGRAM ARRAY STORAGE: ATD (Model 2)

| Array Name | Dimension | Function |
| :---: | :---: | :---: |
| ID | 128 | ID(I) contains the subscript in ID of the parent group from which group I was split. |
| INDEX | 128 | INDEX (I) ccontains the subscript of the input variable used on the parent group when creating group $I$. |
| HI | 128 | HI(I) contains the subscript of one of the groups created by splitting group I. This is the member of the pair with the (algebraically) largest mean. |
| LO | 128 | LO(I) contains the subscript of the other of the groups created by splitting group I. This is the member of the pair with the smaller mean. |
| TN = LAST | 128 | TN(I) contains the number of observations contained in group I. |
| TWT | 128 | TWT (I) contains the sum of weights for the observations in group $I$. |
| TY1 | 128 | TYI (I) contains $\Sigma Y$ for the observations in group I. |
| TY2 | 128 | TY2(I) contains $\Sigma Y^{2}$ for the observations in group I. |
| FAIL $=$ SIGN | 128 | FAIL (I) contains 0 if there has never been a failure to split group I given an attempted partition, it contains a 1 if an attempted partition has failed or if the group has already been split. |
| LOC | 128 | LOC (I) contains the subscript in $D$ of the first observation in the threaded list comprising group I. |
| MEAN | 128 | MEAN(I) contains the mean $Y$ for group i. |

APPENDIX F-- (CONTINUED)

| Array Name | Dimension | Function |
| :--- | :--- | :--- |
| LIST = 64 | Temporary storage used during partitioning. <br> LIST (I) contains the new group number to be <br> assigned to all observations in the parent <br> group for which the predictor used in the <br> partition has the value appearing in the <br> array KODE (I). |  |
| KODE |  |  |

The following arrays correspond exactly to those described above, except they contain the statistics for the best available predictor for partitioning the group under consideration.
(BSSP, BSS), (CODE, KODE), ( $\mathrm{N}, \mathrm{N} 1$ ) , (W,W1), (Y1,Y3), (Y2,Y4), (YBAR,Y5)

| Array Name | Dimension | Function |
| :---: | :---: | :---: |
| P | 36 | Subscripts of predictor variables. |
| NAME 1 | 36 | First word of the alphanumeric name of that predictor |
| NAME2 | 36 | Second word of the alphanumeric name of that predictor |
| TYPE | 36 | Type of predictor, monotonic or free. |
| LAB | 12 | Alphanumeric run identification. |
| CLASS | 256 | Contains partition rule split identification codes. The region is divided up into 128 blocks of two words. The 36 bits in each word are arbitrarily identified as follows. <br> For example: |
|  |  | 0 0 0 0  0 1 1 0 1 0 Word 1 <br> 35 34 33 . . . 4 3 2 1 0 [CLASS (9)] |
|  |  | 0 0 0 0 0 0 0 0 0 0 <br> 636261 • • • 383736 [CLASS (10)] <br> Each pair of words contains information identifying the values of the partition variable used in assigning observations into the group with which that pair of words is associated. If group 5 is created via a split of group 3 such that all observations in group 5 have the values 4 or 1 or 3 on predictor $X_{p}$ used in the split, the words 9 and 10 in ${ }^{P}$ the Array CLASS would look as illustrated. |



## APPENDIX G

Program Write-Up<br>Institute for Social Research The University of Michigan<br>IBM 7090

Program: Function IRFORM
Programmer: T. C. O'Brien
Source Language: UMAP
Date: November 1963
Function:
Much of the data collected by the Institute for Social Research prior to 1959 contain codes not easily capable of being handled by the format statements available in the MAD and FORTRAN II programming languages. Moreover, it is desirable for general purpose library programs to have data-input formats read in at execution time, rather than compiled with the program. It is also desirable for a dictionary of the locations of the input variables on the cards printed out for ease in interpretation of the output, and for ready checking of the correctness of the format statement read in.

This subroutine may be incorporated into any MAD, FORTRAN II or UMAP program compiled or assembled by the translators in the U. of M. Executive System for the IBM 7090. It accomplishes the following:

1. Reads and edits format information punched in columns 1-72 on a series of cards, either MAD or FORTRAN specifiers.
2. Prints out a dictionary of the locations of the variables on the input cards.
3. Supplies the edited format information to the calling program.
4. Edits subsequent data which is read in by the main program, employing several BCD to BINARY conversion schemes not easily available in FORTRAN or MAD.

Input:
Format information presented to IRFORM must be enclosed in parentheses regardless of whether it consists of MAD field specifiers or FORTRAN specifiers. It must be punched in columns 1-72 of any number of consecutive cards. The word "FORMAT" is not punched on the cards, nor are any continuation marks used. Thus, MAD format information is punched (......*).

The following list of FORTRAN field specifiers are permissible:
I, F, A, H, X, E, O
The following FORTRAN operators and symbols are permissible:
P , ( / ) . + - blank
The following MAD field specifiers are permissible:
I, F, C, H, S, E, K
The following MAD operators and symbols are permissible:
P , (/) . + - * blank
Any legal IBM character may occur inside an $H$ string.

These characters and operators, when used in their proper form will be supplied to the program, with the following restrictions:

1. Parentheses may not be nested inside the format statement.
2. NO FIELD except an $H$ string or a series of skip $X$ (FORTRAN) or skip $S$ (MAD) fields may be more than six (6) columns in width.

Several new field specifiers have been established. They have meaning similar to the ones above, except that BCD to BINARY conversion takes place in subroutine IRFORM instead of in the standard system inputoutput subroutines. When a special field indicator is read in, it is replaced as follows and a conversion switch is set.

| Format: | Stored as: |  |
| :--- | :---: | :--- |
| nLw | nAw | BCD to 12-base integer |
| nTw | nAw | BCD to 12-base floating point integer |
| nJw | nAw | BCD to 10-base integer |
| nGw.d | nAwbb | BCD to 10-base floating point number |

(Where $\mathrm{b}=\mathrm{blank}, \mathrm{d}=$ decimal places, n is the field repetition operator and $w$ is the field width in columns.)

Scale factors (P indicators) may not be used with G field specifiers.
The purpose of establishing these format field descriptors is two-fold.

1. The L and T specifiers permit single-punches in rows 12 ( + ) and $11(-)$ of the IBM card to be read into the machine and used as integers. They may be stored in the machine in integer mode, or in floating point mode.
2. The J and G specifiers permit program control of punching patterns in fields which, though legal alphanumeric patterns, result in an I/O dump when read in through $I$ and $F$ field specifications.

In the $L$ and $T$ BCD to BINARY conversion rule, single punched columns in a field have the following internal machine values:

| Card | Value |
| :---: | :---: |
| $0-9$ | $0-9$ |
| + | 10 |
| - | 11 |
| blank | -0 |

Thus, the value of a field read in and converted through an $L$ or $T$ specifier may be represented internally as follows:

$$
a_{1}(12)^{n-1}+a_{2}(12)^{n-2} \ldots+a_{n-1}(12)^{1}+a_{n}(12)^{0}
$$

where $a_{1}$, $a_{2}, \ldots a_{n}$ are the internal machine values of the symbols as defined above and $n$ is the number of colums in the field. Note: (12) ${ }^{0}=1$. Thus, a two-column field goes into the machine as a positive integer in the range from 0 to 143 . Note that all variables entering this way must be positive integers, since the symbols normally used to differentiate between positive and negative numbers are now part of the number system itself. A table of conversions for two-column codes is appended.

In the $J$ and $G$ BCD to BINARY conversion rules, single-punched columms in a field have the following internal machine values:

| Card | Value |
| :---: | :---: |
| $0-9$ | $0-9$ |
| + | -0 |
| - | -0 |
| blank | -0 |

The only difference between the special field designator $J$ and the standard MAD or FORTRAN Integer (I) designator is in the treatment of nonnumeric characters when they appear on the data cards. The same is true of the $G$ designator and the MAD or FORTRAN ( $F$ ) field designators. When a J or G specifier is used, then all character patterns which are illegal in the corresponding $I$ or $F$ field are reduced to the value -0 , rather than causing a halt of the computer, followed by an I/O dump. A flag is then set, which can be interrogated by the main program.

The following rules apply:

1. All data fields read through standard FORTRAN or MAD field specifications are not edited by IRFORM. The FORTRAN and MAD manuals describe what punching patterns may legally appear in these types of fields.
2. For the fields, L, T, J, and G, all illegal character patterns read from data cards result in the internal machine value of the field -0.
3. For the fields, $L, T, J$, and $G$, all characters other than,+- , $0-9$ or blank are illegal, except for decimal points read in through a G field.
4. In addition, for $L$ and $T$ fields, any field containing a blank is illegal. All combinations of the set of characters [0 $\left.\begin{array}{lllllllllll}0 & 2 & 3 & 4 & 5 & 6 & 7 & 9+-\end{array}\right]$ are legal.
5. For $J$ and $G$ fields, the only legal character patterns are
[blank (s)] followed by [at most one sign ( + or - )], followed by at least one digit 0123456789 and continuing with digits to the right end of the field. Items in brackets [ ] are optional. In a $G$ field, a decimal point may appear immediately to the left or to the right of any digit, and take precedence over the number of decimals specified in the format description. An all blank field is illegal.
6. If at least one illegal character pattern is detected when the data are read in through the format statement, a signal is returned to the calling program.

To summarize:
The purpose of the $L$ and $T$ fields is to provide a means of converting nonblank fields containing patterns of single-punches into 12-base integers, under program control, sather than under $I / O$ subroutine control. The purpose of $J$ and $G$ fields is to convert signed or unsigned numbers into decimal numbers in integer or floating point form, providing for the conversion of character patterns in these fields, which are not signed numbers (usually strings of characters, e.g., ( --- or $+{ }_{+1}$, etc.) into a representation usable by the computer, under program control, rather than under $I / O$ subroutine control, since these are illegal and would cause an $I / O$ dump.

## Calling Sequences:

## FORTRAN II

$\mathrm{J}=$ IRFORM (FMT , LEN, ISTART, IEND, EDIT, IDIEN)

MAD
$J=$ IRFORM. (FMT, LEN, ISTART, IEND, EDIT, IDLEN)
where
FMT is the name of the first element of the vector in which the format statement is to be stored. (Mode is Floating in FORTRAN, Integer in MAD)
LEN is the dimensioned length of the array FMT in the calling program. (Integer mode)

ISTART is the number (subscript) to be printed out in the dictionary for the first field read by the format statement. (Integer mode)
IEND is the number (subscript) to be printed out in the dictionary for the last field read by the format statement. (Integer mode)
EDIT is the name of the first element of a one-dimensioned array or vector in the calling program in which format conversion codes will be stored by IRFORM. (Integer in MAD, Floating in FORTRAN)
IDLEN is the dimensioned length of the array EDIT. It should be one element longer than the array $X$ used by the calling program to read in data. (Integer mode)

UMAP examples:

| CALL IRFORM | CALL EDITPM |
| :--- | :--- |
| TXH FMI | TXH X |
| TXH $=200$ | TXH $=500$ |
| TXH $=0$ | TXH EDIT |
| TXH $=50$ |  |
| TXH $=$ EDIT |  |
| TXH $=501$ |  |

$\circ$
0
FMT BES 200
EDIT BES 501
0
0

Subroutine Functioning.
A call to IRFORM causes card images to be read from input tape 7 and the format statement is scanned, edited for special fields and stored away. Special fields are detected, converted to A (character) fields and an entry is made in the vector EDIT for each special field encountered. An exit flag (J) is returned to the calling program. If at least one special field is encountered, $J=1$. J $J=0$ if no special fields were encountered. Card reading and scanning continues until either a zero-level of parentheses (as many left as right) has occurred, or until the dimensioned length of FMI is about to be exceeded, or until the dimensioned length of EDIT is about to be exceeded. If either of the latter conditions occurs, the program cannot continue and a memory dump is initiated. J is returned as a FORTRAN or MAD integer depending on the language of the calling program. No variable which is more than six columns wide may be read in through an $\mathrm{L}, \mathrm{T}, \mathrm{J}$, or G field specifier. Otherwise a dump will result.

The structure of the edit list is as follows: The first word in the list contains the number of elements which follow in the list.

| Prefix | Decrement | Tag | Address |
| :---: | :---: | :---: | :---: |
| 0 | i | w | c |

where $i$ is the index of the variable to be edited in the input region defined by the calling program, $w$ is the field width, and c is a conversion code. The conversion code specifies the input mode, the type of conversion desired and, for $G$ fields, the number of (implied) decimal places in the field. The edit list is used by means of another entry into IRFORM which is executed after each vector of data is read into the computer using FMT'. The function value $J$ as defined above may be used to determine whether or not any special formats were read in, thus requiring an edit on each vector of data. Thus, conversion of data values is accomplished by the following calling sequences:

FORTRAN II

$$
\mathrm{L}=\text { EDITPM. (X, LEN, EDIT) }
$$

MAD
L = EDITPM. (X, LEN, EDIT)
In each case $X$ is the appropriate MAD or FORTRAN base address of an input vector with length dimensioned at LEN and EDIT is defined as above. $X$ may be integer or floating mode, LEN is integer, and EDIT is floating mode in FORTRAN and integer mode in MAD. Execution of this statement causes the necessary transformations to be made on those variables listed as requiring them in the edit list. The results are placed back in the corresponding positions in the $X$ array. If an illegal field has been detected, the value of $L$ is nonzero, otherwise it is zero. L is reset each time EDITPM is executed. $L$ is an integer of the appropriate form returned to the calling program.

A typical FORTRAN code sequence might be as follows:
DIMENSION X (100), EDIT (101), FMT (108)
$\mathrm{NX}=50$
$\mathrm{J}=\operatorname{IRFORM}$ (FMT, 108, 0, NX, EDIT, 101
1 READ INPUT TAPE 7, FMT, CTYPE, (X (I), $\mathrm{I}=1$, NX)
IF (J) 2,5,2
$2 \mathrm{~L}=\operatorname{EDITPM}(\mathrm{X}, 100, \operatorname{EDIT})$
3 IF (L) 4, 5, 4
4 locate undefined value of field.
Some $X_{i}=-0$ and take appropriate action
5 CONTINUE
process data card
6 GO TO 1

L \& T CONVERSION TABLE FOR TWO COLUMN FIELDS

L O W ORDER COLUMN

|  |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | + | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| H | 0 | 00 | 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 09 | 10 | 11 |
| I | 1 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| H | 2 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 |
|  | 3 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 |
| ORDER | 4 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
|  | 5 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 6 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 |
| c | 7 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 |
| 0 | 8 | 96 | 97 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 |
| M |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 9 | 108 | 109 | 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 |
|  | $+$ | 120 | 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 | 131 |
| N | - | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 | 141 | 142 | 143 |



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```
    WHENEVER (NP-IX*6) NE, O, IX=IX+1 AIO20056
    P=IX*6 A1020057
    KONST=IX+2 A102005*3
    WHENEVER(N*KONST) .G. 20000,TRANSFER TO EXIT2 AIDZ0059
R
R SET SWITCHES FOR MISSING OATA.
R
    WHENEVER CAS.(MC2,-0.) .NE. O
    NOMD=3
    DTHERWISE
        WHENEVER CAS.(MD1,-0.) .NE. O
        NOMD=2
        OTHERWISE
        NOMD=1
        END OF CONDITIONAL
    END UF CUNDITIONAL
    QZ=0
    WHENEVER CAS.(YMAX,-0.) .E. O, QZ=1
R
R SET SCALE FACTOR SWITCH.
R
    FY=1.
    SQ=1
    WHENFVER SFA
    SQ=0
    FY=10.0.P.SCFIN
    END OF CONDITIINAL 
R SCALE MD1, MD2, AND YMAX
    TRANSFER TO PASS(SQ)
    WHENEVER MB, MD2=MD2*FY
    WHENEVER MA, MDI=MDI*FY
    WHENEVER YA, YMAX=YMAX*FY
R
PASS(1) OPL=1
    DP2=1
    I D=Y
    INDEX=WT
R
    PRINT FURMAT HEAD, LAB(0)...LAB(12),
1P1,P2,ITRMAX,Y,NAME1,NAME 2, YMAX
    I =NP/4
    WHENEVER (NP-I*4) .NE. O, I=I + I
R
R READ PREDICTORS, TYPES, AND NAMES -- TYPE 4.
R
    THROUGH FEED,FOR J=1, l,J .G. I
    K=(J-1)*4+1
    L=K+3
    READ FORMAT CARD4,TYPE, (M=K,1,M.G.L,P(M),TYPE(M),NAMEI(M),
lNAME2(M))
    WHENEVER TYPE .NE. $4$,TRANSFER TO EXIT3
FEEO
    CONTINUE
```



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NOP(1)
TRA(0) GUOD

HANG(0)
HANG(1)
SKIP(3)
SKIP(2) SKIP(1)

JUMP(0) JUMP (1)

BAD
SKIP(0)
WTAPE(1
WTAPE (0)
ADD(1)

ADD (2)

PACK

LAST
TRANSFER TO BAD
CONTINUE
SIGN=OB
D=V(Y)
TRANSFER TO HANG(SQ)
$D=D * F Y$

R


TRANSFER CHECK VALUE
WHENEVER CAS. (D,MD2).E.O , TRANSFER TO BAD
WHENEVER CAS. (D,MDI).E.O , TRANSFER TU BAD
WHENEVER CAS. (D,-0.).E.O, TRANSFER TU BAD
TRANSFER TO JUMP(OZ)
WHENEVER D.G. YMAX, TRANSFER TO BAD
KARD=\$YES \$
TRANSFER TO SKIP(O)
$R$ NUMBER OF DATA BEING DELETED.
$C D=C O+1$
KARD $=\$ \$$
SIGN=1B
R
TAPE WRITE SWITCH.
TRANSFER TO WTAPE(TW)
WRITE BINARY TAPE BB , KARD, V(1)...V(NV)
WHENEVER SIGN, TRANSFER TD FIRST
R WEIGHT SWITCH
TRANSFER TO ADD(R)
WHENEVER V(WT) .G. 32768, TRANSFER TJ EXIT6
$W H T=V(W T)$
$R$ PUT WEIGHT AND NUMBER EACH DATUM.
$T N=T N+1$
N1 $=$ N1 + KONS $T$
$N 2=N 1+K O N S T$
WHENEVER (N2-1) .G. 20000, TRANSFER TO EXIT2
$X(N 1)=W H T$.V.(N2 .LS. 18$)$
$\mathrm{K}=\mathrm{N} \mathrm{L}+1$
$D(K)=0$
WEIGHT=WHT
TWT = TWT + WEIGHT
$T Y 1=T Y 1+D * W E I G H T$
$T Y 2=T Y 2+D * D * W E I G H T$
R
PACK DATA INTO CORE.
R
THROUGH PACK,FOR $J=1,1, J$.G. $P$
$V=V(P(J))$
WHENEVER $V$.L. 0. UR. $V$.G. 6.3, TRANSFER TO EXITT
WHENEVER $V, G$. $\operatorname{MAX}(J), M A X(J)=V$
WHENEVER (J-J/6*6) -NE. O,TRANSFER TO PACK
$K=K+1$
$x(K)=V(P(J)) \cdot V \cdot(V(P(J-5)) \cdot L S \cdot 6) \cdot V \cdot(V(P(J-4)) \cdot L S \cdot 12)$
1.V.(V(P(J-3)).LS.18).V.(V(P(J-2)).LS.24).V.(V(P(J-1)).LS. 30)A1D20333 CONTINUE TRANSFER TU FIRST
R
PRINT COMMENT 60 DATA ARE ALL IN. $\$$ $R$ READ INPUT BEING こUMPLETED. TRANSFER TO REWINO(TW)
$R$

A1020234
A1020285
AID20286
A1020287
A1020288
A1D20289
AID20290
A1D20291
AID20292
AID20293
AID20294
AID20295
AID20296
AID20297
A1020298
A1020299
AID20300
AID20301
A1020302
AID20303
A1020304
AI020305
AID20306
AID20307
A1020308
AID20309
AI020310
AID20311
AIO20312
A1020313
AID20314
AID20315
A1D20316
A1D20317
A1D20318
A1020319
AID20320
A1D20321
A1D20322
AID20323
A1020324
A1D20325
AID20326
AI020327
AIU20328
A1D20329
A1020330
AID20331
AID20332
AID20334
AID20335
A1020336
A1D20337
A 1020335
A1020339
A1020340

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R
A1020455
VECTOR VALUES YCARD $=\$ 1 H$, S4,I5,30H TH DATA DELETED. VARIABAID20456

```
    LLE(,I4, 5H ) = ,E15.8*S AIO20457
```

    R
    AID20458
    VECTOR VALUES OATA \(=\mathbf{\$ C A R D} \$, \$ T A P E \$\)
    R
    VECTOR VALUES BAKA \(=\$\) NOT \(\$, \$\) \$
        R
    VECTDR VALUES OUT \(=\$\) NUNE \(\$, \$\) CARD \(\$, \$\) TAPE \(\$\), \(\$\) BDTH \(\$\)
    AIO20459
    N1D20460
    A1D20461
    R
    1 OF INTERVAL FROM, I6,3H TO,I6,12H ON VARIABLE,IG/1H, 54,CG, AID20466
    210H WHICH LIE,C6,21HSIDE OF INTERVAL FROM,I6,34 TO,I6,12H ON AIO20467
    3VARIABLE,16*\$
    A1D20468
    R
    AID?0469
        VECTOR VALUES HEAD3=\$1HO,S4,14HRESIDUALS ARE, 66,33 R REOUESTEAID20470
        ID AND OUTPUT WILL BE ,C6,1H.*S AID20471
        \(R\) AID?0472
        VECTOR VALUES \(B O B=1 B\)
        R
    VECTOR VALUES RUN=OB
    AID20473
        R
            ENO OF PROGRAM
    SASSEMBLE, PUNCH UBJECT
IRFORMOLAID20478
CODE MACRD a
2ET HALF
TRA INSERT
STO TEMPC
CLA A
TRA CODESV
CODE END
ENTRY IRFORM
IRFDRM SXA IDXI,1
SXA IDX2,2
SXA $1 D \times 4,4$
STZ RESULT
CLA 1,4
STA STDAWY
AXT 0,1
STZ MAD
CLA* 2,4
PAX ,2
TXH $\quad+3,2,0$
PDX , 2
STL MAD
SXD LENTST,2
CLA\# 3,4
PDX ,2
TXH *2,2,0
PAX , 2
pXA , 2
STO VARINO
STU VARINI
CLA* 4.4
PDX ,2
TXH $\quad+2,2,0$
P』X , 2 .
A1020474
AID20475
A1020476
A1020477
A1020479
A1D20480
AID20481
A1D20482
AID20483
AID20484
AID20485
A1020436
AID20437
AID20488
AID20489
AID 20490
AID20491
A1020492
AID20493
A1020494
Al020445
AID?0496
A1020447
A1020498
AID20499
AlD20500
AIU20501
A1020502
A1020503
AI020504
AID20505
AID20506
AID20507
AID2050
A1020511

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|  | TRA | INREC |  |  |  | A1020569 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RIGHT | CLA | COUNT |  |  |  | A1020570 |
|  | SUB | $=1$ |  |  |  | A1020571 |
|  | TZE | OUT. |  |  |  | A1020572 |
|  | STO | COUNT |  |  |  | AID20573 |
|  | TRA | I NREC |  |  |  | A1020574 |
| OUT | CLA | INPUT, 2 |  |  |  | A1020575 |
|  | STO* | STOAWY |  |  |  | A1020576 |
| LENTST | TXL | OUTPUT, 1,** |  |  |  | A1D20577 |
|  | CALL | SPRINT |  |  |  | AID20578 |
|  | RLK | LENERR, , 4 |  |  |  | AID20579 |
|  | CALL | SYSTEM |  |  |  | A1020580 |
| LENERR QUTPUT | BCI | 4, FDRMAT, IS TO | LONG |  |  | AID20581 |
|  | S×O | ENDTST, 1 |  |  |  | AID20582 |
|  | CALL | SPRINT |  |  |  | AID20583 |
|  | BLK | HEAD, 8 |  |  |  | A1020584 |
|  | CLA | $=1$ |  |  |  | AID20585 |
|  | STO | COLUMN |  |  |  | AID20586 |
|  | STO | CARDNO |  |  |  | A1020587 |
|  | TSX | CARDHO, 1 |  |  |  | AID20588 |
| BACIN | $\triangle X T$ | $0 \cdot 1$ |  |  |  | AIO20589 |
|  | $A X T$ | 6,4 |  |  |  | AID20590 |
|  | LOO* | STOAWY |  |  |  | AID20591 |
|  | STZ | INT |  |  |  | AID20592 |
| OPI | ZAC |  |  |  |  | AID20593 |
|  | LGL | 6 |  |  |  | A1020594 |
|  | CAS | TEN |  |  |  | A1020595 |
|  | TRA | FILDTP |  |  |  | A1020596 |
|  | NOP |  |  |  |  | AID20537 |
|  | STO | TEMP |  |  |  | AID20598 |
|  | CLA | INT |  |  |  | AID20599 |
|  | ALS | 2 |  |  |  | A1020600 |
|  | ADD | INT |  |  |  | A1020601 |
|  | ALS | 1 |  |  |  | AID20602 |
|  | $\triangle C L$ | TEMP |  |  |  | A1020603 |
|  | STO | INT |  |  |  | AID20604 |
| NXTCHR | TSX | INCRE, 2 |  |  |  | A1020605 |
|  | TRA | OPI |  |  |  | AID20506 |
| $\begin{aligned} & \text { HEAD } \\ & \text { CARDHD } \end{aligned}$ | BCI | 8,1 VARIABLE | NUMBER | COLUMNS | TYPE | AID20607 |
|  | SXA | IDX1H, 1 |  |  |  | AIO20608 |
|  | SXA | IDX2H,2 |  |  |  | A1020609 |
|  | SXA | IDX4H,4 |  |  |  | AID 20610 |
|  | CLA | CARDNO |  |  |  | AID20611 |
|  | TSX | CONVTI, 2 |  |  |  | AID20612 |
|  | STO | CDHEAD+1 |  |  |  | AID20613. |
|  | CALL | SPRINT |  |  |  | A1020614 |
|  | BLK | CDHEAD, , 2 |  |  |  | A1020615 |
| $10 \times 1 \mathrm{H}$ | $\triangle X T$ | **, 1 |  |  |  | A1020616 |
| $1 \mathrm{D} \times 2 \mathrm{H}$ | $A X T$ | **, 2 |  |  |  | A1020617 |
| $10 \times 4 \mathrm{H}$ | $A X T$ | **,4 |  |  |  | AID20618 |
|  | TRA | 1,1 |  |  |  | A1020619 |
| CDHEAD | BCI | 2,OCARO |  |  |  | AID20620 |
| CONVTI | SXA | IDX1I,1 |  |  |  | A1020621 |
|  | SXA | 10x2I,2 |  |  |  | AID20622 |
|  | SXA | 10x41,4 |  |  |  | A1020623 |
|  | $A X T$ | 0,4 |  |  |  | AID20624 |
|  | LRS | 35 |  |  |  | AID20625 |

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|  | CLA | ELANKS | AID206,26 |
| :---: | :---: | :---: | :---: |
|  | STO | BUILD | A1D20627 |
| CONT 1 | TXI | * + 1, 4, 1 | 41020528 |
|  | CLM |  | AID 20629 |
|  | DVP | TEN | 41020630 |
|  | SLW | TEMPI, 4 | A1020631 |
|  | STO | TEMP11 | A1020632 |
|  | CLA | TEMP I 1 | A1020633 |
|  | TNZ | CONTI | A1020634 |
|  | CAL | BUILD | A1020635 |
|  | ALS | 6 | AID20536 |
|  | ORA | TEMPI,4 | AID20637 |
|  | TIX | *-2,4,1 | A1D20638 |
|  | SLW | BUILD | AID20639 |
|  | CLA | BUILD | AID20640 |
| IDX1I | AXT | **, 1 | AlD20641 |
| $10 \times 21$ | $A X T$ | **, 2 | 41020642 |
| I $0 \times 41$ | $\wedge \times T$ | **, 4 | 11020643 |
|  | TRA | 1,2 | A1020644 |
| TEMPI 1 | PZE |  | AIO20645 |
| TEMPI | BES | 5 | AID20646 |
| I NCRE | TIX | EOTST $2,4,1$ | AID20647 |
|  | TXI | * $+1,1,1$ | AID20648 |
| ENDTST | TXL | EDTST1, $1, * *$ | AID20649 |
|  | LXA | VARINO, 2 | AID? 0650 |
| LSTSTL | TXH | FINSH, 2,** | AID? Oós |
|  | ZET | HALF | A1020652 |
|  | TRA | IOXI | A1D20653 |
|  | CLA | CARDNO | AID20654 |
|  | $\triangle D D$ | $=1$ | A IC20555 |
|  | STO | CARDNO | A10206.56 |
|  | TSX | CARDHD, 1 | A1020657 |
|  | CLA | $=1$ | A1020658 |
|  | STO | COLUMN | A1020659 |
|  | LXA | GROUP1,2 | A1020660 |
|  | TRA | RESTOR | $\triangle 1020661$ |
| EDTSTI | LDQ* | STOAWY | A [120662 |
|  | $A \times T$ | 6,4 | A1D20663 |
| EDTST2 | TRA | 1,2 | A 1020564 |
| FILDTP | $\Delta \times T$ | TABLEN, 2 | A1020665 |
|  | CAS | TAB+1,2 | A D 20566 |
|  | TRA | + +2 | A ID20か67 |
|  | TRA* | SWITCA+1,2 | AID20568 |
|  | TIX | FILDTP $+1,2,1$ | $\triangle 1020659$ |
|  | TRA | ERR | AID20s70 |
|  | BCI | 1,000001 | AID20671 |
|  | BC I | 1,00000K | A10?067? |
|  | BCI | 1,00000F | A1020673 |
|  | BCI | 1.00000E | A1E20674 |
| A | BCI | 1,00000A | A1020675 |
|  | BCI | 1,00000H | 41020076 |
|  | BCI | 1.00000C | A 1020677 |
|  | BCI | 1,00000S | A1020678 |
|  | BCI | 1,000000 | A1D20679 |
|  | $B C I$ | 1,00000 T | AID20680 |
|  | BCI | 1,00000L | A17200*al |
|  | BCI | 1,00000, | A10206s? |

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|  | TRA | LINFL2 | A 1020911 |
| :---: | :---: | :---: | :---: |
| ERR | CALL | SPRINT | AID20912 |
|  | BLK | ERCOM, 3 | AID20913 |
|  | CALL | SYSTEM | AID20914 |
| I D $\times 1$ | $A X T$ | * * 1 | AIO20915 |
|  | CLA | TABPOT | AID20916 |
|  | AXT | 0.4 | AID20917 |
|  | ZET | MAD | AID20918 |
|  | SSM |  | A1020919 |
|  | STO* | TABPDT | AID20920 |
|  | CLA | RESULT | AID20921 |
| $1 \mathrm{DX2}$ | $A X T$ | **.2 | AID20922 |
| I DX4 | $\triangle X T$ | **, 4 | AID20923 |
|  | TRA | 2,4 | AID20924 |
| READ | TSX | /TV/SCARDS,4 | AID20925 |
|  | BLK | INPUT, , EDF | AID20926 |
|  | TRA | 1.2 | AID20927 |
| EOF | TSX | /TV/SPRINT,4 | AID20928 |
|  | BLK | EOFCM, 3 | A1020929 |
|  | TSX | /TV/SYSTEM,4 | AID20930 |
| EOFCM | BCI | 3, END OF FILE. | AID20931 |
| ERCOM | BCI | 3, ILLEGAL CHARACTER | AID20932 |
| VARINO | PZE |  | AID20933 |
| COLUMN | PZE |  | AID20934 |
| CARDNO | PLE |  | A1020935 |
| TEN | PLE | 10 | AID20936 |
| INT | PZE |  | AID20937 |
| TEMP | PZE |  | AID20938 |
| BUILD | PZE |  | AID20939 |
| FIELD | PLE |  | AID20940 |
| REPFLD | PZE |  | A ID20941 |
| GROUP 1 | PZE |  | AID20942 |
| GROUP | PZE |  | AIO20943 |
| MQTEM | PZE |  | AID20944 |
| TEMPC | PLE |  | AID20945 |
| SAVMQ | PZE |  | AID20946 |
| LINE | BCI | 7, | AID20947 |
| MINUS | BCI | 1. | AID20948 |
| BLANK 7 | BCI | 1, 0 | AID20949 |
| COUNT | PZE |  | AID20950 |
| LPARN | BCI | 1,000001 | AID20951 |
| BLANKS | BCI | 1. | AID20952 |
| RPARN1 | BCI | 1,00000) | AID20953 |
| UNDWD | MZE |  | A1020954 |
| VARINI | PZE |  | AID20955 |
| SPECFD | PZE |  | AID20956 |
| CODEWD | PZE |  | AID20957 |
| BLCHK | PZE |  | AID20958 |
| RESULT | PZE |  | AID20959 |
| MAD | PZE |  | AID20960 |
| INTFLT | PLE |  | AID20961 |
| DWIOT 1 | PZE |  | AID20962 |
| VALUE | PZE |  | AID20963 |
| DGSW | PZE |  | AID20964 |
| SIGNSW | PZE |  | AID20965 |
| MZE | MZE |  | A1D20966 |
| MASKT | OCT | 700000 | A1020967 |

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| INPUT | BSS | 30 | A1020968 |
| :---: | :---: | :---: | :---: |
| FILDWT | PZE |  | A1020969 |
| FildC | CODE | $=2$ | AID20970 |
| FILDCL | CODE | $=1$ | A1020971 |
| FILDCJ | CODE | $=3$ | A1D20972 |
| FILDCG | CODE | $=10$ | AID20973 |
| CODESV | STO | CODEWO | A 1020974 |
|  | STL | SPECFD | A1020975 |
|  | STL | RESULT | A1D20976 |
|  | TRA | FILDCH +1 | A1020977 |
| FINSH | ZET | HALF | AID20978 |
|  | TRA | IDXI | A1020979 |
|  | STL | HALF | AI020980 |
|  | CLA | VARIN1 | AID20981 |
|  | Sto | varino | AID20982 |
|  | tra | BACIN | AID20983 |
| INSERT | LDQ* | StOAWY | A1D20984 |
|  | STZ | BLCHK | AI 020985 |
|  | ZAC |  | AI 020986 |
|  | SXD | + 2,4 | AID20987 |
|  | AXT | 6,2 | A1020988 |
| INSETI | TXL | INSET2,2,** | AID20989 |
|  | LGL | 6 | AID20990 |
|  | TXI | INSET1,2,-1 | AID20991 |
| INSET2 | ALS | 6 | AID20992 |
|  | $A C L$ | A | A ID20993 |
|  | SLW | temp | AID20994 |
|  | LGL | 6 | A 1020995 |
|  | TNX | INSET3,2,1 | AID20996 |
| INSETS | CAL | TEMP | A 1020997 |
|  | ALS | 6 | AID20998 |
|  | SLW | TEMP | AID20999 |
|  | ZAC |  | A 1021000 |
|  | LGL | 6 | A ID21001 |
|  | CAS | BLANK | A ID21002 |
|  | TRA | INSET8 | A ID21003 |
|  | TRA | INSET6 | AID21004 |
|  | CAS | TEN | A1D21005 |
|  | TRA | INSET4 | A1D21006 |
|  | NOP |  | A1021007 |
| [NSET7 | zet | BLCHK | AID21008 |
|  | CLA | BLANK | AID 21009 |
| I NSET6 | ACL | TEMP | A1D21010 |
|  | SLW | TEMP | AID21011 |
|  | TIX | INSET5,2,1 | AID21012 |
| INSET3 | CAL | TEMP | AID21013 |
|  | SLW* | Stoany | AID21014 |
|  | CLA | BLANKS | A1D21015 |
|  | Sto | TEMP | AID21016 |
|  | TXI | * $+1,1,1$ | A1021017 |
|  | AXT | 6,2 | 4ID21018 |
|  | LDO* | Stoany | A1D21019 |
|  | TRA | INSET5 | AID21020 |
| BLOUT | STL | BLCHK | AID? 1021 |
|  | TRA | INSET7 | A1D21022 |
| INSET4 | CAS | ABRPN | AID21023 |
|  | TRA | INSETA | AID21024 |

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$\left.\begin{array}{lll} & & \\ & \text { TRA } & \text { INSET8 }\end{array}\right]$ AID21025

| SWITCH | NZ $T$ | MAD | INTEGER | AID21082 |
| :---: | :---: | :---: | :---: | :---: |
|  | TRA | STOWY |  | A1021083 |
|  | ALS | 18 |  | AID21084 |
| Stowy | STO* | PICKUP |  | AID21085 |
|  | TXI | * $+1,2,1$ |  | AID21086 |
| TSTEND RETURN | TXL | AROUND, 2,** |  | AID21087 |
|  | CLA | RESULT |  | AID21088 |
|  | LXA | IDX1, 1 |  | AID21089 |
|  | rRA | ID×2 |  | AID21090 |
| FLOAT | TXH | *+2, 1, 0 |  | AID21091 |
|  | LXA | OWIDTH,1 |  | A1D21092 |
|  | ORA | FTCONT |  | A1021093 |
|  | FAD | FTCONT |  | A1021094 |
|  | FDP | TABP, 1 |  | A 1021095 |
|  | XCA |  |  | AID21096 |
|  | TRA | STOWY |  | AID21097 |
|  | DEC | 1.0E6 |  | AID21098 |
|  | DEC | $1.0 E 5$ |  | AID21099 |
|  | DEC | 1.0 E 4 |  | AlD21100 |
|  | DEC | 1.0 E 3 |  | AID21101 |
|  | DEC | 1.0 E 2 |  | A1021102 |
|  | DEC | 1.0El |  | A1D21103 |
| t ABP | DEC | 1. |  | A1D21104 |
| FTCONT | DEC | 155B8 |  | AID21105 |
| GCONV | STZ | INTFLT |  | A1D21106 |
|  | TRA | + +2 |  | AID21107 |
| JCONV | STL | INTFLT |  | AID21108 |
|  | STZ | OWIDTI |  | A1021109 |
|  | ANA | MASKT |  | AID21110 |
|  | ALS | 3 |  | AID21111 |
|  | PDX | , 1 |  | AID21112 |
|  | STZ | value |  | A1D21113 |
|  | Stz | TEMP |  | AID21114 |
|  | STZ | DGSW |  | AID21115 |
|  | STZ | SIGNSW |  | 41D21116 |
| NXTDIG | ZAC |  |  | AID21117 |
|  | LGL | 6 |  | AID21118 |
|  | CAS | blank |  | 41021119 |
|  | TRA | UNDEFV |  | 41021120 |
|  | TRA | NXTCHI |  | A1021121 |
|  | CAS | MINUSP |  | AID21122 |
|  | tra | UNDEFV |  | A1D21123 |
|  | TRA | MSIGN |  | AID21124 |
|  | CAS | PLUS |  | A1021125 |
|  | TRA | DPOINT |  | AID21126 |
|  | tra | PSIGN |  | A1021127 |
| NXTCH3 | CAS | $=10$ |  | AID21128 |
|  | TRA | UNDEFV |  | AID21129 |
|  | TRA | UNDEFV |  | AID21130 |
|  | STA | TEMP |  | AID21131 |
|  | STL | DGSW |  | AID 21132 |
|  | CLA | value |  | A1D21133 |
|  | ALS | 2 |  | A1021134 |
|  | ADD | value |  | A1D21135 |
|  | ALS | 1 |  | AID21136 |
|  | ADD | temp |  | AID21137 |
|  | STO | value |  | AID21138 |

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| NXTCH2 | Tix | NXTDIG,1,1 | A1021139 |
| :---: | :---: | :---: | :---: |
|  | NZT | DGSW | AID21140 |
|  | TRA | UNDEFV | A1021141 |
|  | LXA | DWIDT1,1 | A1021142 |
| NXTCH4 | 2ET | INTFLT | AID21143 |
|  | TRA | SWITCH | AID21144 |
|  | TRA | float | AID21145 |
| MSIGN | 2ET | SIGNSW | A1021146 |
|  | TRA | UNDEFV | AID21147 |
|  | STL | SIGNSW | A1021148 |
|  | CLA | MZE | AID21149 |
|  | STO | value | AID21150 |
|  | Sto | TEMP | AID21151 |
|  | ZET | DGSW | AID21152 |
|  | TRA | UNDEFV | AID21153 |
|  | TRA | NXTCH2 | AID21154 |
| PSIGN | ZET | SIGNSW | AID21155 |
|  | TRA | UNDEFV | AID21156 |
|  | ZET | OGSW | A1021157 |
|  | TRA | UNDEFV | AID21158 |
|  | STL | SIGNSH | A1021159 |
|  | TRA | NXTCH2 | AID21160 |
| NXTCHI | ZET | DGSW | AID21161 |
|  | TRA | UNDEFV | AID21162 |
|  | ZET | SIGNSW | A1D21163 |
|  | TRA | UNDEFV | AID 21164 |
|  | TRA | NXTCH2 | AIO21165 |
| DPOINT | cas | POINTD | A1021166 |
|  | tra | UNDEFV | AID21167 |
|  | tra | - +2 | AID21168 |
|  | TRA | NXTCH3 | A1D21169 |
|  | TXI | *+1,1,-1 | A1021170 |
|  | SXA | OWIDT1,1 | A1D21171 |
|  | TXI | NXTCH2,1,1 | AID21172 |
| TCONV | STz | INTFLT | A1021173 |
|  | tra | $\cdots+2$ | AID21174 |
| LCONV | STL | INTFLT | AID21175 |
|  | ANA | MASKT | A1021176 |
|  | ALS | 3 | A1021177 |
|  | PDX | , 1 | A1D21178 |
|  | STZ | value | AID21179 |
|  | STZ | TEMP | A1021180 |
|  | STz | SIGNSW | A1021181 |
| NXICLT | ZAC |  | AID21182 |
|  | LGL | 6 | AID21183 |
|  | cas | BLANK | AID21184 |
|  | TRA | UNDEFV | AID21185. |
|  | tra | LTCONI | AID21185 |
|  | CAS | MINUSP | AID21187 |
|  | tra | UNDEFV | A1021188 |
|  | TRA | EM | AID21189 |
|  | CAS | PLUS | AID21190 |
|  | tra | UNDEFV | A1021191 |
|  | tra | EP | AID21192 |
|  | CAS | $=10$ | AID21193 |
|  | TRA | UNDEFV | AID21194 |
|  | TRA | UNDEFV | AID21195 |


| LTCON2 | Sta | TEMP | A1021196 |
| :---: | :---: | :---: | :---: |
|  | CLA | value | AID21197 |
|  | ALS | 1 | AID21198 |
|  | ADD | value | AID21199 |
|  | ALS | 2 | AID21200 |
|  | ADD | TEMP | AID21201 |
|  | Sto | value | AID21202 |
|  | STL | SIGNSW | AID21203 |
| LTCON3 | TIX | NXTCLT, 1,1 | A1D21204 |
|  | NZ T | SIGNSW | AID21205 |
|  | TRA | UNDEFV | A1021206 |
|  | TXI | NXTCH4, 1, -1 | AID2 1207 |
| EP | CLA | $=10$ | A ID2 1208 |
|  | tra | LTCON2 | AID2 1209 |
| EM | CLA | =11 | AID21210 |
|  | tra | LTCON2 | AID21211 |
| LTCON1 | 2E.T | SIGNSW | AID21212 |
|  | TRA | UNDEFV | AID21213 |
|  | TRA | LTCON3 | AID21214 |
| UNDEFV | CLA | UNDWD | AID21215 |
|  | STL | RESULT | AID2 1216 |
|  | TRA | STOWY | AID21217 |
| TABERR | CALL | SPRINT | AID21218 |
|  | BLK | TABERC, 3 | AID21219 |
|  | CALL | ERROR | A1021220 |
| WDERR | CALL | SPRINT | A1D21221 |
|  | BLK | WDERRC, 6 | A1021222 |
|  | CALL | ERROR | AID21223 |
| SAVER | CALL | SPRINT | AID21224 |
|  | BLK | SAVERC., 4 | 41021225 |
|  | CALL | ERROR | AID21226 |
| WDERRC | BCI | 6, FIELD WIDTH MORE THAN 6 | AID21227 |
| SAVERC | BCI | 4, EDIT TABLE EXCEEDED | AID21228 |
| TABERC | BCI | 3, BAD EDIT TABLE | AID21229 |
| MINUSP | SYN | TAB | AID21230 |
|  | END |  | AID21231 |
| SASSEMB | be, P | CH OBJECT | SAPTIMOIAID21232 |
|  | ENTRY | WRATIM | AID21233 |
| SAVE | PZE |  | A1D21234 |
|  | PZE |  | AID21235 |
|  | PZE |  | AID21236 |
| SIXTY | DEC | 60 | AID21237 |
| HRS | PZE |  | AID21238 |
| MIN | PZE |  | AID21239 |
| SEC | PZE |  | AID21240 |
| FRACT | P2E |  | AID21241 |
| WRATIM | SXD | SAVE,4 | AID21242 |
|  | S×D | SAVE+1,2 | AID21243 |
|  | $5 \times 0$ | SAVE $+2,1$ | AID21244 |
|  | CALL | DAYtim | AID21245 |
|  | LRS | 35 | AID21246 |
|  | DVP | SIXTY | AID21247 |
|  | Sto | FRACT | AIDP 1248 |
|  | $2 A C$ |  | AID21249 |
|  | DVP | SIXTY | AID21250 |
|  | STO | SEC | AID21251 |
|  | ZAC |  | A1021252 |




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```
    R SAVE THE BEST SPLIT INFORMATION. AID2148I
    R
    WHENEVER TMAX .L. BMAX
    TMAX=BMAX
    SAVE=CHECK
    PX=JP
    PV=JI
    PMAX=SMAX
    THROUGH REMA,FOR I=0,I,I .G. SAVE
    J=CODE(C(I))
            Nl(I)=N(J)
            Wl(I)=W(J)
                    Y3(I)=Y1.(J)
                    Y4(I)=Y2(J)
                    Y5(I)=Y8AR(I)
                    KODE(I)=J
                    BSSP(I)=BSS(I)
    BSSP(64)=BSS(64)
    END OF CONDITIONAL
CHOICE
CONTINUE
R
R END OF PARTITION SCAN.
R TEST IF SPLIT SATISFIES CRITERIOV 2.
R
    WHENEVER TMAX.LE. PB
    SIGN(PG)=1B
    PRINT FORMAT OUT5, PG,PX,TMAX
    SIGN=1B
    TRANSFER TO SEARCH
    END OF CONDITIONAL
R
R PERFORM PARTITION - ASSIGN SPLIT GROUP I.D.S. AIDZ15L2
R
    NOGP=NOGP + 2
    WHENEVER NOGP.G. 127, TRANSFER TO EXIT
    GA=NOGP - L
    GB=NOGP
    N=0
    W=0.0
    Y1=0.0
    Y2=0.0
R
R STORE PARTITION CODES -- FIRST GRDUP.
R
    THROUGH ONE,FOR K=0,1,K .G. PMAX
    I =KODE (K)
    WHENEVER I -L. 36
    CLASS(GA,1)=(1 .LS. l) .V. CLASS(GA,1)
    OTHERWISE
    CLASS(GA,2)=(1.LS.(I-36).V.CLASS(GA,2)
    END DF CONDITIONAL
    LIST(I)=GA
    N=N+N1(K)
    Yl=Yl +Y 3(K)
    Y2=Y2+Y4(K)
    W=W+W1(K)
    N(1)=0
    TMAX=BMAX TMAX . - BMAX 
    AID21482
    AID21483
    AID21484
    AID21485
    AID21486
    A1021487
REMA
ONE
```





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R AID21709
VECTOR VALUES OUTI=\$1H,S4,16H FOR VARIABLE,I4,3H 1,2C6, AID21710 12 H ),11H B S $S=$ E15.8,S8, 11H BSS/TSS = F8.5*\$ AID21711
R ENO OF PROGRAM AID21712
END OF PROGRAM AID21713
\$ASSEMBLE, PUNCH OBJECT
ENTRY WRATIM
SAVE PZE
PZE
PZE

| HRS | PZE |
| :--- | :--- |
| MIN | PZE |

SEC PZE
FRACT PZE
WRATIM SXD SAVE,4
SXD SAVE+1,2
SXD SAVE+2,1
CALL DAYTIM
LRS 35
DVP SIXTY
STO FRACT
ZAC SIXTY
STO SEC
ZAC SIXTY
STO MIN
ZAC SIXTY
STO HRS
PRINT FMA,HRS,MIN,SEC,FRACT,O
LXD SAVE,4 AID21741
LXD SAVE+1,2
LXD SAVE+2,1
TRA 2,4
FMA BCI *,12HOTIME IS NOW,4(I3,1H.)*
END
\$BREAK
AID21719
AID21720
A1021721
AID21722
AID21723
AID21724
A1D21725
AID21726
AID21727
AID21728
AID21729
AID21730
AID21731
AID21732
AID21733
AID21734
A1021735
AID21736
AID21737
AID21738
A1021739
AID21740
$A 1021741$
AID21742
AID21743
AID21744
A1D21745 AID21746 AID21747
s COMPILE MAD,PRINT OBJECT,PUNCH OBJECT

R
R
R
$R$
$R$
$R$

PROGRAM NAME -- A I D. THIRD CDRE.
WRITTEN BY ROBERT W HSIEH.
AIO - MODEL 2 - REWRITTEN ON

RIMENSIDN ID(128),INDEX(128)。H1(128),LO(128), TN(128), TWT(128)A1021755
$2, P(36), T S S(128), B S S(128), M E A N(128), N(128), C(72), T Y P E(36), A I D 21757$
3 NAME1(36), NAME2(36),X(20000),D(20000),V(100),LAB(12) AID21758
R
PROGRAM COMMON NAME1, NAME2, NP, NV, LAB, A1021759 1 MAX, P, NOGP, ITR, ITRMAX, PA, PG,OPI, OP2,X,MSIZE,SCFIN, SCFOUT,
2 KONST, AA, BB, RUN, ZWANT, ZTYPE, ZTAPE,BOB, TYPE
R
EQUIVALENCE $(K, K L),(X, D),(I, L),(S C F O U T, S F B)$


```
    WHENEVFR C.NE. O, PRINT FORMAT OUT4, C(I)...こ(C) A1D21823
    Q=TYI(I) AMEAN(I) AID21824
    WHENEVER HI(I) .E. O AID21825
    \(N G=N G+1.0\)
    PRINT CDMMENT\& ** THIS GROUP IS RETAINED AS DNE UF AID2IB28
1 FINALS. \(\$\)
    END UF CONDITIONAL
    D=MEAN(I)-MEAN(1)
    TSS(1)=TY2(I)-G
    BSS(I)=SQRT. (.ABS.(TSS(I)/TINT(I)))
    RI =TWT(I)/TWT(I)*100.0
    R2 \(2=T S S(I) / T S S\)
    \(B S S=B S S+0\)
    AID21826
    AID21827
    AID21828
    AID21829
    AID21830
    AID21831
    AID21832
    A1021833
    AIO21834
    AIO21835
    PRINT FORMAT OUT5,TN(I),MEAN(I), D,TYIIII,TWT(I),BSS(II,TSS(I)AID21836
1 ,TY2(I),R1,Q,R2 AID21837
    CONTINUE A1D21838
\(R\) AID21839
\(R \quad\) PRINT ANALYSIS UF VARIAVCE TABLE. AID21840
R PRINT COMMENT AID21841
    PRINT COMMENT\$4 ***
IVARIANCE TABLF
                MEAiNs.
                            VARIATIUN SQUARES
                            SQUARE
                                F\$
1 FREEDOM
    BSS=BSS-TSS(1)
    \(D=T S S-B S S\)
    RI = BSS/NG
    \(Q=N-N G\)
    \(R 2=0 / Q\)
    MEAN=R1/R2
    PRINT FORMAT ABC, TSS,N,BSS,NG,R1,MEAN,D,Q,R2
R
\(R\) COMPUTE RESIDUALS.
R
    TRANSFER TO MIS(ZWANT)
R
\(R\) IDENTIFY EACH DATUM WITH ITS GROUP NUMBER.
    WHENEVER HI(I) •E.O
    \(X=\operatorname{LOC}(1)\)
    \(J=X(X), R S .18\)
    \(X(X)=I\)
    WHENEVER J .E. O, TRANSFER TO BBC
    \(X=J\)
    TRANSFER TU ABCD
    END OF CONDITIONAL
R
    REWIND TAPE 4
    REWIND TAPE 3
R
    \(K=1-K\) ONS T
    \(N N=N V+1\)
    \(V=1\)
R
\(R\) SET SCALE FACTOR SWITCH.
```

SOURCF UF
MIS(1) THROUGH BBC, FOR $I=2,1, I . G$. NOGP

MIS(1)
$A B C D$

BBC

```
WHENEVER \(C\) NE 0 PRINT FORMAT OUT4, C(1)...e(C)
```

```
    -1D21837
    AID21842
    AID21843
    AID21844
    A1021845
    AID21846
    AID21847
    AID21-64
    AID21849
    A1D21850
    AID21851
    AID21852
    AID21853
    AID21854
    A1D21855
    A1D21855
    AID? 1857
    A1D21858
    A1021859
    A1021860
    A1021861
    AID21862
    AI021853
    AID21864
    A1021865
    AID21366
    A1021867
    AID21868
    AID21869
    AID21870
    AIO21871
    A1021872
    A1021873
    A1D21874
    AID21875
    AID21876
    AID21877
    AID21878
    AID21879
```




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7190111525000915
71901215300009 ..... 16
71902111350010 ..... 17
71902211450008 ..... 18
71902311601008 ..... 19
71902112350010 ..... 20
71902212450009 ..... 21
71902312601012 ..... 22
71902322202009 ..... 23
71902322200009 ..... 24
71902113350010 ..... 25
71902213450011 ..... 26
71902313600011 ..... 27
71902114350010 ..... 28
71902214450012 ..... 29
71902314600009 ..... 30
71902115250011 ..... 31
71902215300011 ..... 32
71903111500010 ..... 33
71903211550010 ..... 34
71903311701010 ..... 35
71903112500010 ..... 36
71903212550010 ..... 37
71903312700008 ..... 38
71903113503010 ..... 39
71903213550010 ..... 40
71903313699010 ..... 41
71903223200010 ..... 42
71903223202010 ..... 43
71903114503010 ..... 44
71903214550010 ..... 45
71903314700012 ..... 46
71903115250009 ..... 47
71903215300009 ..... 48
71904111579008 ..... 49
71904211620010 ..... 50
71904311801010 ..... 51
71904112580012 ..... 52
71904212620010 ..... 53
71904312801010 ..... 54
71904113581008 ..... 55
71904213630010 ..... 56
71904313800010 ..... 57
71904114580012 ..... 58
71904214630010 ..... 59
71904314800010 ..... 60
71904224202011 ..... 61
71904224200011 ..... 62
71904115250011 ..... 63
71904215300010 ..... 64
71905111570010 ..... 65
71905211640010 ..... 66
71905311960010 ..... 67
71905112580010 ..... 68
71905212650010 ..... 69
71905312960010 ..... 70
71905113560010 ..... 71
000000000111111111122222222233333333334444444444555555555566666666667777777777812345678901234567890123456789012345678901234567890123456789012345678901234567890
71905213650010 ..... 72
71905313950010 ..... 73
71905114570010 ..... 74
71905214660010 ..... 75
71905314950010 ..... 76
71905124200012 ..... 77
71905124202012 ..... 78
71905115250010 ..... 79
71905215300011 ..... 80
71906111101012 ..... 81
71906311100012 ..... 82
71906112100011 ..... 83
71906212101011 ..... 84
71906113101010 ..... 85
71906213100010 ..... 86
71906114100009 ..... 87
71906224100008 ..... 88
71906215101009 ..... 89
71906125101008 ..... 90E
2130

## APPENDIX I

## ON TRANSFERRING AID (2) TO ANOTHER COMPUTER

The program was written for a 32 k IBM 7090 with an on-line clock which is interrogated by the program. The U.M. 7090 has a core-protect device. Any transfer to another computer will have to take these factors into account. In general, there will be few problems with tape limitations on other equipment, since the program uses only five tapes as follows: $B C D$ input, $B C D$ output, two scratch tapes and one program segmentation (ping-pong) tape.

The program will run in its present form on any 32 k IBM 709 or 7090 computer capable of accepting the University of Michigan Executive (MAD) system, September 1963 version.

The program is written in MAD (not Fortran) and uses several subroutines written in IMAP, in addition to input-output and other subroutines supplied by the Executive System.

Thus, if the potential user has access to an IBM 709 or 7090 system and if his computing center administration can operate, at least part of the time, under the U. of M. Executive System, the present MAD program may be used. Since the program is primarily written in MAD, it cannot be used in its present form on a computer which does not have a MAD translator implemented for it. This would require re-writing the source program in FORTRAN, ALGOL or some other suitable language. This can be done, but would require considerable programming skill, and a knowledge of both MAD and FORTRAN or ALGOL. It is estimated that an equivalent FORTRAN program would be somewhat larger than the MAD program.

However, complete documentation in the form of descriptions of storage allocation, flow charts, listings, etc., are provided in this document.

A potential user should:

1. Determine from his computing center whether it has the University of Michigan Executive System available for use, and if not, whether it can be obtained. (It is available from the IBM user's organization called SHARE.)
2. If this is the case, an IBM 1401 -compatible tape ( $1 / 2$ inch, 200 or 556 bpi density) may be shipped to:

Data Processing Section
Institute for Social Research
The University of Michigan
Box 1248
Ann Arbor, Michigan 48106
together with a request for AID 2. Desired tape density must be specified. A small charge will be made to cover handling and shipping costs. The symbolic program, and test data will be written on tape and shipped as unblocked 80 -character BCD records in the desired density.
3. If the necessary equipment or Executive System is not available, the documentation presented here should be sufficient for conversion of the program to another computer of suitable size.

For either use, the following materials would be useful:

1. University of Michigan Executive System for the IBM 7090 Computer, University of Michigan Computing Center, September 1963.
2. Michigan Algorithmic Decoder, Bruce Arden, Bernard Galler, Robert Graham, University of Michigan Computing Center, January 1963.

## Disclaimer

Although this program has been tested thoroughly by its programmer, no warranty, express or implied, is made by the programmer or the Institute for Social Research or the University of Michigan as to the accuracy and functioning of the program and related program material, and no responsibility is assumed by the programmer, the Institute or the University in connection therewith.

# APPENDIX J 

# PROBLEMS IN THE ANALYSIS OF SURVEY DATA, AND A PROPOSAL*** 

James N. Morgan and John A. Sonquist*<br>University of Michigan


#### Abstract

Most of the problems of analyzing survey data have been reasonably well handled, except those revolving around the existence of interaction effecta. Indeed, increased efficiency in handling multivariate analyses even with non-numerical variablea, has been achieved largely by assuming additivity. An approach to survey data is proposed which imposes no restrictions on interaction effects, focuses on importance in reducing predictive error, operates sequentially, and is independent of the extent of linearity in the classifications or the order in which the explanatory factors are introduced.


## A. NATURE OF THE DATA AND THE WORLD FROM Which they COME

Tนе increasing availability of rich data from cross section surveys calls for more efficient methods of data scanning and data reduction in the process of analysis. The purpose of this paper is to spell out some of the problems arising from the nature of the data and the nature of the theories which are being tested with the data, to show that present methods of dealing with these problems are often inadequate, and to propose a radical new method for analyzing survey data. There are seven things about the data or about the world from which they come which need to be kept in mind.
First, there is a wide variety of information about each person interviewed in a survey. This is good, because human behavior is motivated by more than one thing. But the very richness of the data creates some problems of how to handle them.

Second, we are dealing not with variables for the most part, but with classifications. These vary all the way from age, which can be thought of as a variable put into classes, to occupation or the answers to attitudinal questions, which may not even have a rank order in any meaningful sense. Even when measures seem to be continuous variables, such as age or income, there is good reason to believe that their effects are not linear. For instance, people earn their highest incomes in the middle age ranges. Expenditures do not change uniformly with changes in income at either extreme of the income scale.

Third, there are errors in all the measures, not just in the dependent variable, and there is little evidence as to the size of these errors, or as to the extent to which they are random.

Fourth, the data come from a sample and generally a complex one at that. Hence, there is sample variability piled on top of measurement error. The fact that almost all survey samples are clustered and stratified leads to problems of the proper application of statistical techniques. Statistical tests usually assume simple random samples rather than probability samples. More ap-

[^5]propriate tests have been developed for simple statistics such as proportions, means, and a few others.
Fifth, and extremely important, there are intercorrelations between many of the explanatory factors to be used in the analysis-high income goes along with middle age, with advanced education, with being white, with not being a farmer, and so forth. This makes it difficult to assess the relative importance of different factors, since their intercorrelations get in the way. Since many of them are classifications rather than continuous variables, it is not even easy to measure the extent of the intercorrelation. Measures of association for cross classification raise notoriously difficult problems which have not really been solved in any satisfactory way. ${ }^{1}$
Sixth, there is the problem of interaction effects. Particularly in the social sciences, there are two powerful reasons for believing that it is a mistake to assume that the various influences are additive. In the first place, there are already many instances known of powerful interaction effects-advanced education helps a man more than it does a woman when it comes to making money; and it does a white man more good than a Negro. The effect of a decline in income on spending depends on whether the family has any liquid assets which it can use up. Women have their hospitalizations at different ages than men. Second, the measured classifications are only proxy variables for other things and are frequently proxies for more than one construct. Several of the measured factors may jointly represent a theoretical construct. We may have interaction effects not because the world is full of interactions, but because our variables have to interact to produce the theoretical constructs that really matter. The idea of a family life cycle, unless arbitrarily created out of its components in advance, is a set of interactions between age, marital status, presence, and age of children. ${ }^{2}$ It is therefore often misleading to look at the over-all gross effects of age or level of education. Where interaction effects exist, the concept of a main effect is meaningless, and it is our belief that in human behavior there are so many interaction effects that we must change our approach to the problems of analysis.
Another example of interaction effects appeared in the attempt to build equivalent adult scales to represent the differences in living expenses of families of different types. After many years of analysis, one of the most recent studies in this field has concluded "when its size changes, families' tastes appear to change in more complicated ways than visualized by our hypothesis."3 More

[^6]recently in analyzing factors affecting spending unit income, it has become obvious that age and education cannot operate additively with race, retired status, and whether the individual is a farmer. The attached table illustrates this with actual average incomes for a set of nonsymmetrical groups. The twenty-one groups account for two-thirds of the variance of individual spending unit incomes, whereas assuming additivity for race and labor force status even with joint age-education variables produces a regression which with 30 variables accounts for only 36 per cent of the variance. A second column in the

TABLE 1. SPENDING UNIT INCOME AND THE NUMBER IN THE UNIT WITHIN VARIOUS SUBGROUPS

| Group | Spending unit average (1958) income | Number in unit | Number of cases |
| :---: | :---: | :---: | :---: |
| Nonwhite, did not finish high school | 82489 | 3.3 | 191 |
| Nonwhite, did finish high school | 5005 | 3.4 | 67 |
| White, retired, did not finish high school | 2217 | 1.7 | 272 |
| White, retired, did finish high achool | 4520 | 1.7 | 72 |
| White, nonretired farmers, did not finish high school | 3950 | 3.6 | 87 |
| White nonretired farmers, did finish high school | 6750 | 3.6 | 24 |
| The Remainder |  |  |  |
| 0-8 grades of achool |  |  |  |
| 18-34 years old | 4150 | 3.8 | 72 |
| 35-54 years old | 4670 | 3.8 | 240 |
| 55 and older-not retired | 4846 | 2.2 | 208 |
| 9-11 grades of school |  |  |  |
| 18-34 years old | 5032 | 3.7 | 112 |
| 35-54 years old | 6223 | 3.4 | 202 |
| 55 and older-not retired | 4720 | 2.1 | 63 |
| 12 grades of achool |  |  |  |
| 18-34 years old | 5458 | 3.3 | 193 |
| 35-54 years old | 7765 | 3.8 | 291 |
| 55 and older-not retired | 6850 | 2.0 | 46 |
| Some college |  |  |  |
| 18-34 years old | 5378 | 3.0 | 102 |
| 35-54 years old | 7930 | 3.8 | 112 |
| 55 and older-not retired | 8530 | 2.0 | 36 |
| College graduatea |  |  |  |
| 18-34 years old | 7520 | 3.8 | 80 |
| 35-54 years old | 8566 | 2.9 | 150 |
| 55 and older-not retired | 10879 | 1.8 | 34 |

[^7]table gives the average number of people in the unit, and it can be seen that this particular breakdown is not particularly useful for analyzing the number of people in a unit. On the other hand, if each group were to be used to analyze expenditure behavior, income, and family size are likely to operate jointly rather than additively.

In view of the fact that intercorrelation among the predictors on the one hand and interaction effects on the other are frequently confused, it seems uscful to give a pictorial example indicating both the differences between them and the way in which they operate when both are present. Our concern is not with statistical tests to distinguish between them, but with the effects of ignoring their presence.

Chart I shows pictorially three cases, real but exaggerated. First, there is a case where the two explanatory factors, income and education, are correlated with one another, but do not interact. Second, a case where income and being self-employed interact with one another but are not correlated, and third, a situation where income and asset holdings are correlated with one another and also interact in their effect on saving. The ellipsoids represent the area where most of the dots on a scatter diagram would appear. In the first case, it is clear that, a simple relation between income and saving would exaggerate the effect of income on saving by failing to allow for the fact that high income people have more education, and that highly educational people also save more. An ordinary multiple regression, however, using a dummy variable representing high education would adequately handle this difficulty. In the second case there is no particular correlation, we assume, between income and being selfemployed, but the self-employed have a much higher marginal propensity to save than other people. Here, the simple relationship between income and saving becomes a weighted compromise between the two different effects that really exist. A multiple correlation would show no effect of being self-employed and the same compromise effect of income. Only a separate analysis for the self-employed and the others would reveal the real state of the world. In the third case, not only do the high-asset people have a higher marginal propensity to save, but they also tend to have a higher income. Multiple correlation clearly will not take care of this situation in any adequate way. It will produce an "income effect" which can be added to an "asset effect" to produce an estimate of saving. Here the income effect is an average of two different income effects. The estimated asset effect is likely to come out closer to zero than if income had been ignored. Of course, where interactions exist, there is little use in attempting to measure separate effects.

Finally, there are logical priorities and chains of causation in the real world. Some of the predicting characteristics are logically prior to others in the sense that they can cause them but cannot be affected by them. For instance, where a man grows up may affect how much education he gets, but his education cannot change where he grew up. We are not discussing here the quite different analysis problem where the purpose is not to explain one dependent variable but to untangle the essential connections in a network of relations.

In dealing with a single dependent variable representing some human behavior, we might end up with at least three stages in the causal process-early
childhood and parental factors, actions and events during the lifetime, and current situational and attitudinal variables. If this were the end of the problem we could simply run three separate analyses. The first would analyze the effects of early childhood and parental factors. The second would take the residuals from this analysis and analyze them against events during a man's lifctime up until the present, and the third would take the residuals from the

Muticollinearity, i.e., correlation between income and education but no interaction


Interaction, but no multicollinearity (no correlotion between income and self -employment)



Chart I. Combinations of Multicollinearity and Interaction and Their Effects.
second analysis and analyze them against current situational and attitudinal variables. But the real world is not even that simple, because some of the same variables which are logically prior in their direct effects may also tend to mediate the effect of later variables. For instance, a man's race has a kind of logical priority to it, but at the same time it may affect the way other things such as the level of his education operate to determine his income.

This is an impressive array of problems. Before we turn to a discussion of current attempts to solve these problems and to our own suggestions, it is essential to ask first what kind of theoretical structure is being applied and what the purposes of analysis are.

## B. NATURE OF THE THEORY AND PURPOSES OF ANALYSIS

Perhaps the most important thing to keep in mind about survey data in the social sciences is that the theoretical constructs in most theory are not identical with the factors we can measure in the survey. The simple economic idea of ability to pay for any particular commodity is certainly a function not only of income but of family size, other resources, expected future income, economic security, and even extended family obligations. A man's expectations about his own economic future, which we may theorize will affect his current behavior, might be measured by a battery of attitudinal and expectational questions or by looking at his education, occupation, age, and the cxperience of others in the same occupation and education group who are already older. The fact that the theoretical constructs in which we are interested are not the same as the factors we can measure, nor even simply related to thom, should affect our analysis techniques and focus attention on creating or locating important interaction effects to represent these constructs.

Second, there are numerous hypotheses among which a selection is to be made. Even if the researcher preferred to restrict himself to a single hypothesis and test it, the intercorrelations among the various explanatory factors mean that the same result might support any one of several hypotheses. ${ }^{4}$ Hence, comparisons of relative importance of predictors, and selecting those which reduce predictive errors most, are required.

When we remember that there are also variable errors of measurement, the problem of selecting between alternative hypotheses becomes doubly difficult, and ultimately requires the use of discretion on the part of the researcher. Better measurement of a factor might increase its revealed importance.

Finally, researchers may have different reasons why they wish to predict individual behavior. Most will want to predict behavior of individuals in the population, not just in the sample, which makes the statistical problem somewhat more complicated. But some may also want to focus on the behavior of some crucial individuals by assigning more weight to the behavior of some rather than others. Others may want to test some explanatory factors, however small their apparent effect, because they are important. They may be important because they are subject to public policy influences or because they

[^8]are likely to change over time, or because they are crucial to some larger theoretical edifice. The nature of these research purposes thus combines with the nature of the data and their characteristics to make up the problem of how to analyze the data.

## C. THE STRATEGY CHOICE IN ANALYSIS

One can think of a series of strategies ranging from taking account of only the main effects of each explanatory classification separatcly or jointly, to trying to take account of all possible combinations of all the classifications at once. Even if there were enough data to allow the last, however, it would not be of much use. The essence of research strategy then consists of putting some restrictions on the process in order to make it manageable. One possibility is to cut the number of explanatory factors utilized, and another is to restrict the freedom with which we allow them to operate. ${ }^{\text {s }}$ One might assume away most, or all interaction effects, for instance, and keep a very large number of explanatory classifications. Still further reduction in the number of variables is possible, if one assumes linearity for measured variables or, what amounts to the same thing builds arbitrary scales, incestuously derived out of the same data in order to convert each classification into a numerical variable. Clcarly, the more theoretical or statistical assumptions one is willing to impose on the data, the more he can reduce the complexity of the analysis. A difficulty is that restrictions imposed in advance cannot be tested. There seems some reason to argue that it would be better to use an approach which developed its restrictions as it went along. In any case keeping these problems in mind we turn now to a summary of how analysis problems in using survey data are currently being handled and some of the difficulties that present methods still leave unsolved.
D. HOW PHOBLEMS IN ANALYSIS ARE CURRENTLY BEING HANDLED-AN APPRAISAL

We take the seven problems in section $A$ in the same order in which they are presented there plus the major problem in section $B$, that of theoretical constructs not measured directly by the factors on which we have data. The first problem was the existence of many factors. The simplest procedure has been to look at them one at a time always keeping in mind the extent to which one factors is intercorrelated with others. Another technique, particularly with attitudes, has been to build indexes or combinations of factors either arbitrarily or with the use of some sort of factor analysis technique. ${ }^{6}$ The difficulty is that the first of these is quite arbitrary, and the second is arbitrary in a different sense, in that most mechanical methods of combining factors are based on the intercorrelations between the factors themselves and not in the way in which they may affect the dependent variable. It is quite possible for two highly correlated factors to influence the dependent variable in opposite ways. Building a combination of the two only on the basis of their intercorrelation would create a factor which would have no correlation at all with the dependent

[^9]variable. With highly correlated attitudes, however, some such reduction to a few factors may be required and meaningful.

With the advent of better computing machinery, the problem of multiple factors has frequently been handled by using multiple correlation techniques. The use of these techniques, of course, required solving the second problem, that arising from the fact that in many cases we have classifications rather than continuous variables. This has been done in two ways, first, by building arbitrary scales. For instance, one could assign the numbers one, two, three, four, five, and six to the six age groups in order. Or if age were being used to predict income, one could assign a set of numbers representing the average income of people in those age groups. ${ }^{7}$ But unless machine capacity is extremely limited, a far more flexible method which is coming into favor is to use what have been called dummy variables. ${ }^{\text {T }}$ The essence of this technique is to assign a dummy variable to each class of a characteristic except one. It is called a dummy variable because it takes the value one if the individual belongs in that subclass or a zero if he does not. If ordinary regression procedures are to be used, of course, dummy variables cannot be assigned to every subclass of any characteristic, since this would overdetermine the system. However, at the Survey Research Center we have developed an iterative program for the IBM 7090, the output of which consists of coefficients for each subclass of each characteristic, the set for each characteristic having a weighted mean of zero. This means that the predicting equation has the over-all mean as its constant term, and an additive adjustment for each characteristic, depending on the subclass into which the individual falls on that characteristic. This is the standard analysis of variance formulation when all interactions are assumed to be zero. Of course, the coefficients of dummy variables using a regular matrix inversion routine can easily be converted into sets of this sort. There remain two difficulties with this technique. One is the problem of interaction effects, which are either assumed away or have to be built in at the beginning in the creation of the classes. A second arises from the nature of the classifications frequently used in survey data. Even though association between, say, occupation and the incidence of unemployment faced by an individual is not terribly high, the occupation code generally includes one or two categories such as the farmers and the retired who, by definition, cannot be unemployed at all. When dummy variables are assigned to these classes, it may easily occur that there is a perfect association between a dummy variable representing one of these peculiar (not applicable) groups in one code and a dummy variable representing something else in another classification (not unemployed). If the researcher omits one of each such pair of dummy variables in a regression routine, he is all right.

A third problem, that of errors in the data, is generally handled by not re-

[^10]jecting hypotheses too easily and by attempting to use some judgment in the assessment of relative importance of different factors or different hypotheses keeping in mind the accuracy with which the variables have probably been measured.

The fact that the data come from a sample has frequently been ignored. As the analysis techniques become more complicated, it becomes almost impossible to keep the structure of the sample in mind too. However, there is some reason to believe that the clustering and stratification of the sample become less and less important the more complex and more multivariate the analysis being undertaken. ${ }^{9}$

What about intercorrelations among the predictors? The main advantage of multivariate techniques like multiple regression is that they take care of these intercorrelations among the predictors, at least in a crude sense. Indeed, if one compares an ordinary subclass mean with the multivariate coefficient of the dummy variable associated with belonging to that subclass, the difference between the two is the result of adjustments for intercorrelations. Where these differences seem likely to be the result of a few major interrelations, some statement as to the factors correlated with the one in question (and responsible for the attenuation of its effect on the multivariate analysis) are often given to the reader. It is, of course, true that where intercorrelations between two predictors are too high, no analysis can handle this problem, and it becomes necessary to remove one of them from the analysis.

Perhaps the most neglected of the problems of analysis has been the problem of interaction effects. The reason is very simple. The assumption that no interactions exist generally leads to an extremely efficient analysis procedure and a great reduction in the complexity of the computing problem. Those of us who have looked closely at the nature of survey data, however, have become increasingly impressed with the importance of interaction effects and the uscful way in which allowing for interactions between measured factors gets us closer to the effects of more basic theoretical constructs. Where interaction effects have not been ignored entirely, they have been handled in a number of ways. They can be handled by building combination predictors in the first place, such as combinations of age and education or the combination of age, marital status, and children known as the family life cycle. ${ }^{10}$ Sometimes where almost all the interactions involve the same dichotomy, two separate analyses are called for. ${ }^{11}$ Interactions are also handled by rerunning the analysis for

[^11]some subgroup of the population. In a recent study of factors affecting hourly earnings, for instance, the analysis was rerun for the white, nonfarmer males only, to test the hypothesis that some of the effects like that of education were different for the non-whites, women, and farmers. ${ }^{12}$ A difficulty with this technique, of course, is that if one merely wants to see whether the interaction biases the estimates for the whole population seriously, one reruns the analysis with the group that makes up the largest part of the sample. But if one wants to know whether there are different patterns of effects for some small subgroup, the analysis must be run for that small subgroup.

Another method of dealing with interaction effects is to look at two- and three-way tables of residuals from an additive multivariate analysis. This requires the process, often rather complicated and expensive, of creating the residuals from the multivariate analysis and then analyzing them separately. ${ }^{13}$ Where some particular interaction is under investigation, an effective alternative is to isolate some subgroup on a combination of characteristics such as the young, white, college graduates. It is then possible to derive an estimate of the expected average of that subgroup on the dependent variable by summing the multivariate coefficients multiplied by the subgroup distributions over each of the predictors. Comparing this expected value with the actual average for that subgroup indicaies whether there is something more than additive effect. It is only feasible to do this with a few interactions, just as it is possible to put in cross product terms in multiple regressions in only a few of the total possible cases. Consequently, most of these methods of dealing with interaction effects are either limited, or expensive and time-consuming.

Still another technique for finding interactions is to restrict the total number of predictors, use cell means as basic data, and use a variance analysis looking directly for interaction effects. ${ }^{14}$ Aside from the various statistical assumptions that have to be made, this turns out to be a relatively cumbersome method of dealing with the data. It requires a good deal of judgment in the selecting of the classes to avoid getting empty cells or cells with very small numbers of cases,

[^12]and the unequal cell frequencies lead to heterogeneity of variances which makes the $F$-test nonconservative. Sometimes interaction effects are considered important only when they involve one extremely important variable. In the case of much economic behavior, current income appears to be such a variable. In this case one can rely on covariance techniques, but these techniques tend to become far too complex when a large number of other factors are involved. Also, as more and more questions arise about the meaning of current income as a measure of ability to pay, the separation of current income for special treatment becomes more doubtful.

Finally, it is also true that if we restrict the number of variables, multiple regression techniques, particularly using dummy variables, can build in almost all feasible interaction effects. One way to restrict the number of variables is to make an analysis with an initial set and run the residuals against a second set of variables. However, unless there is some logical reason why one set takes precedence over another, this is treacherous since the explanatory classifications used in the second set will have a downward bias in their coefficients if they are at all associated with the explanatory classifications used in the first set. ${ }^{16}$

All these methods for dealing with interaction effects require building them in somehow without knowing how many cases there are for which each interaction effect could be relevant. The more complex the interaction, the more difficult it is to tell, of course.

The problem of logical priorities in the data and chains of causation can be handled either by restricting the analysis to one level or by conducting the analysis sequentially, always keeping in mind that the logically prior variables may have to be reintroduced in later analyscs on the chance that they may mediate the effects of other variables. In practice, very little analysis of survey data has paid much attention to this problem. Perhaps the reason is that only recently has anyone been able to handle the other problems so that a truly multivariate analysis was possible. And it is only when many variables begin to be used simultaneously that the problem of their position in a causal structure becomes crucial.

Finally, there is the problem remaining from section B that the constructs of theories do not have any one-to-one correspondence with the measures from the survey. Sometimes this problem is handled by building complex variables that hopefully represent the theoretical construct. The life cycle concept, for instance, has been used this way. In a recent study, a series of questions that seemed to be asking evaluations of occupations were translated into a measure which was (hopefully) an index measure of achievement motivation. ${ }^{16}$ More commonly, the analyst has been constrained to interpret each of the measured characteristics in terms of some theoretical meaning which it hopefully has. This is often not very satisfactory. In the case of liquid assets, the amount of

[^13]these assets a man has represents both his past propensity to save and his present ability to dissave, two effects which could be expected to operate in opposite directions. In general, the analysis of survey data has been much better than this summary of problems would indicate. Varied approaches have been ingeniously used, and cautiously interpreted.

## E. PROPOSAL FOR A PROCESS FOR ANALYZING DATA

One way to focus on the problems of analyzing data is to propose a better procedure. The proposal made here is essentially a formalization of what a good researcher does slowly and ineffectively, but insightfully on an IBM sorter. With large masses of data, weighted samples, and a desire for estimates of the reduction in error, however, we need to be able to simulate this process on large scale computing equipment. The basic idea is the sequential identification and segregation of subgroups one at a time, nonsymmetrically, so as to select the set of subgroups which will reduce the error in predicting the dependent variable as much as possible relative to the number of groups. A subgroup may be defined as membership in one or more subclasses of one or more characteristics. If more than one characteristic is used, the membership is joint, not alternative.

It is assumed that where the problem of chains of causation and logical priority of one variable over another exists, that this problem will be handled by dividing the explanatory variables or predictors into sets. One then takes the pooled residuals from an analysis using the first set of predictors and analyses these residuals against the second set of predictors. The residuals from the analysis using this second set could then be run against a third set. In practice, we might easily end up with three states-early childhood or parental factors, actions and events during the lifetime, and current situational and attitudinal variables.

The possibilities of interactions between variables in different stages can be handled by reintroducing in the second or third analyses, factors whose simple effects have already been removed, but which may also mediate the effects of factors at one of the later stages, that is, nonwhites may have their income affected by education differently from whites.

Temporarily setting aside these complications, we turn now to a description of the process of analysis using the variables from any one stage of the causal process. Since even the best measured variable may actually have nonlinear effects on the dependent variable, we treat each of the explanatory factors as a set of classifications. As we said, our purpose is to identify and segregate a set of subgroups which are the best we can find for maximizing our ability to predict the dependent variable. We mean maximum relative to the number of groups used, since an indefinitely large number of subgroups would "explain" everything in the sample. To be more sophisticated, if we use a model based on the assumption that we want to predict back to the population, there is an optimal number of subgroups. However, as an approximation we propose that with samples of two to three thousand we arbitrarily segregate only those groups, the scparation of which will reduce the total error sum of squares by at
least one per cent and do not even attempt further subdivision unless the group to be divided has a residual error (within group sum of squares) of at least two per cent of the total sum of squares. This restricts us to a maximum of fifty-one groups. It is just as arbitrary as the use of the 5 per cent level in significance tests and perhaps should be subject to later revision on the basis of experience.

We now describe the process of analysis in the form of a series of decision rules and instructions. We think of the sample in the beginning as a single group. The first decision is what single division of the parent group into two will do the most good. A second decision has then to be made: Which of the two groups we now have has the largest remaining error sum of squares, and hence should be investigated next for possible further subdivision? Whenever a further subdivision of a group will not reduce the unexplained sum of squares by at least one per cent of the total original sum of squares, we pay no further attention to that subgroup. Whenever there is no subgroup accounting for at least two per cent of the original sum of squares, we have finished our job. We turn now to a more orderly description of this process.

1) Considering all feasible divisions of the group of observations on the basis of each explanatory factor to be included (but not combinations of factors) find the division of the classes of any characteristic such that the partitioning of this group into two subgroups on this basis provides the largest reduction in the unexplained sum of squares.

Starting with any given group, and considering the various possible ways of splitting it into two groups, it turns out that a quick examination of any possible subgroup provides a rapid estimate of how much the error variance would be reduced by segregating it:

The reduction in error sum of squares is the same size (opposite sign) as the increase in the explained sum of squares.

For the group as a whole, the sum of squares explained by the mean is

$$
\begin{equation*}
N \bar{X}^{2}=\frac{\left(\sum X\right)^{2}}{N} \tag{1}
\end{equation*}
$$

and the total sum of squares (unexplained by the mean) is

$$
\begin{equation*}
\sum(X-\bar{X})^{2}=\sum X^{2}-\frac{\left(\sum X\right)^{2}}{N} \tag{2}
\end{equation*}
$$

If we now divide the group into two groups of size $N_{1}$ and $N_{2}$ and means $\widehat{X}_{1}$ and $\bar{X}_{2}$, what happens to the explained sum of squares?

$$
\begin{equation*}
\text { Explained sum of squares }=N_{1} \bar{X}_{1}^{2}+N_{2} \bar{X}_{2}^{2} . \tag{3}
\end{equation*}
$$

The division which increases this expression most over $N \bar{X}^{2}$ clearly does us the most good in improving our ability to predict individuals in the sample.

Fortunately we do not even need to calculate anything more than a term involving the subgroup under inspection, since $N$ and $\sum X$ remain known and constant throughout this search process.

$$
\begin{gather*}
N_{2}=N-N_{1}  \tag{4}\\
\sum X_{2}=\sum X-\sum X_{1}  \tag{5}\\
\therefore \text { explained sum of squares }=N_{1}\left(\frac{\sum X_{1}}{N_{1}}\right)^{2}+\left(N-N_{1}\right)\left(\frac{\sum X_{2}}{N-N_{1}}\right)^{2} \\
=\frac{\left(\sum X_{1}\right)^{2}}{N_{1}}+\frac{\left(\sum X-\sum X_{1}\right)^{2}}{N-N_{1}}
\end{gather*}
$$

The number of cases (or proportion of sample) and the sum of the dependent variable for any subgroup are enough to estimate how much reduction in error sum of squares would result from separating it from the parent group.

If it seems desirable, a variance components model which takes account of the fact that we really want optimal prediction of members of the population not merely of the sample, can be used. Indeed, the expression for the estimate of the explained, or "between" component of variance in the population turns out to be

$$
\dot{\sigma}_{B}^{2}=\frac{\left[\frac{N-1}{N-2}\left[\frac{\left(\sum X_{1}\right)^{2}}{N_{1}}+\frac{\left(\sum X-\sum X_{1}\right)^{2}}{N-N_{1}}\right]-\frac{\sum X^{2}}{N-2}\right]-\frac{\left(\sum \mathrm{X}\right)^{2}}{\mathrm{~N}}}{N-\frac{N_{1}{ }^{2}+N_{2}{ }^{2}}{N}}(7)
$$

which, though it looks formidable, contains only one new element and that is a term from the total sum of squares of the original group which is constant and can be ignored in selecting the best split. The expression in the brackets is the explained sum of squares already derived. $N, \sum X$, and $\sum X^{2}$ are known and constant. The denominator is an adjustment developed by Ganguli for a bias arising from unequal $N$ 's. Where $N_{1}$ equals $N_{2}$, the denominator becomes equal to $N_{1}$. The more unequal the $N$ 's, the smaller the denominator, relative to an arithmetic mean of the $N$ 's. The ratio of the explained component of variance to the total is rho, the intraclass correlation coefficient. Hence, in using a population model, we are searching for the particular division of a group into two that will provide the largest rho. ${ }^{17}$ Computing formulas for weighted data or a dummy (one or zero) dependent variable can be derived easily.
(2) Make sure that the actual reduction in error sum of squares is larger than one per cent of the total sum of squares for the whole sample, i.e., $>.01\left(\sum X^{2}\right.$, $-N \bar{X}^{2}$ ) (If not select the next most promising group for search for possible subdivision, etc.)
(3) Among the groups 80 segregated, including the parent, or bereft ones, we now select a group for a further search for another subgroup to be split off. The selection of the group to try is on the basis of the size of the unexplained

[^14]sum of squares within the group, or the heterogeneity of the group times its size, which comes to the same thing. It may well not be the group with the most deviant mean.
In other words, among the groups, select the one where
$$
\sum X_{i j}^{2}-N_{i} \bar{X}_{i}^{2} \text { is largest. }
$$

If it is less than two per cent of the total sum of squares for the whole sample, stop, because no further subdivision could reduce the crror sum of squares by more than two per cent. If it is more than two per cent, repeat Step 1.
Note that the process stops when no group accounts for more than two per cent of the error sum of squares. If a group being scarched allows no further segregation that will account for one per cent, the next most promising group is searched, because it may still be possible that another group with a smaller sum of squares within it can be profitably subdivided.
Since only a single group is split off at a time, the order of scamning to select that one should not affect the results. Since an independent scanning is done each time, the order in which groups are selected for further investigation should not matter either, hence our criterion is a pure efficiency one.
Chart II shows how the process suggested might arrive at a set of groups approaching those given carlier in Table 1. The numbers are rough estimates from Table 1.

## Note on A mount of Detail in the Codes

The search for the best single subgroup which can be split of involves a complete scanning at each stage of each of the explanatory classifications, and within each classification of all the feasible splits. This is not so difficult as it scems, for within any classification not all possible combinations of codes are feasible. If one orders the subclasses in ascending sequence according to their means (on the dependent variable), then it can be shown that the best single division-the one which maximizes the explained sum of squares-will never combine noncontiguous groups.

Hence, starting at either end of the ordered subgroups, the computer will sequentially add one subgroup after another to that side and subtract it from the other side, always recomputing the explained sum of squares. By "explained" we mean that the means of the two halves are used for predicting rather than the over-all mean. Whenever the new division has a higher explained sum of squares, it is retained, otherwise the previous division is remembered. But in any case, the process is continued until there is only one subgroup left on the other side, to allow for the possibility of "local maxima."

The machine then remembers the best split, and the explained sum of squares associated with it, and proceeds to the next explanatory characteristic. If upon repeating this procedure with the subclasses of that characteristic, a still larger explained sum of squares is discovered, the now split on the new characteristic is retained and the less adequate one dropped.

The final result will thus be the best single split, allowing any reasonable


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combination of subclasses of a single category, to maximize the explained sum of squares. It is easy to see that this choice will not depend on the order in which factors are entered, but may depend on the amount of detail with which they are coded. The number of subclasses probably should not vary too much from one factor to the next.
The authors are planning to try out such a program under a grant from the National Science Foundation. Data which have already been analyzed using dummy variable multiple regressions will be re-analyzed to see whether the new program provides new insights.

## DISCUSSION

What is the theoretical model behind this process? Instead of simplifying the analysis by arbitrary or theoretical assumptions that restrict the number of variables or the way in which they operate, this process essentially restricts the complexity of the analysis by insisting that there be a large enough sample of any particular subgroup so that we can be sure it matters, and by handling problems one at a time. This is essentially what a researcher does when first investigating a sample using a sorter and his own judgment. It is assumed that the sample being used in a situation like this is a representative probability sample of a large important population. It is possible that there may be subgroups of the population whose behavior is of more importance than that of other subgroups, in which case it would be easily possible to weight the data to take account of this fact. It may be that there are certain crucial characteristics, the importance of which must be investigated. In this case, either lower admission criteria could be used or an initial arbitrary division of the sample according to this characteristic could be made before starting.

Why not take all possible subsets, in other words, all possible combinations of characteristics, and then start combining subcells where the means are close to one another? The simple reason is that there are far too many possible subsets, and since this is a sample, the means of these subsets are unstablc and unreliable estimatcs. It is true, however, that this is the only way one would avoid all possibility of failing to discover interaction effects. Let us take a simple example of a stituation where the method we propose would fail to discover interaction effects. Suppose we have males and females, old and young, in the following proportions who go to the hospital each year, young females eight per cent, young males two per cent, old females two per cent, old males eight per cent. Assuming half the population is male and half the population is old, the old-young split would give means of five and five per cent, and the malefemale split would give means of five and five per cent. Thus we would never discover that it is young females and old males who go to the hospital. One way out of this difficulty which would also vastly increase the efficiency of the machine processes would be to set up a relatively arbitrary division of the sample into perhaps ten groups to start with, groups which are known to be important and suspected to be different in their behavior. The only problem with this is that the remaining procedures will not be invariant with respect to which initial groups were selected.

Onc can never be sure that there does not exist previous work relevant to any "new" idca. William Belson has suggested a sequential, nonsymmetrical division of the sample which he calls "biological classification," for a different purpose, that of matching two groups on other characteristics used as controls so that they can be compared. ${ }^{18}$ His procedure is restricted to the case where the criterion can be converted to a one-zero division, and the criterion for subdivision is the best improvement in discrimination. The method takes account of the number of cases, i.e., focuses on improvement in prediction, not on levels of significance. We have proposed this same focus. No rules are provided as to when to stop, or in what order to keep searching, though an intelligent researcher would intuitively follow the rules suggested herc.
Another approach to the problem as been suggested and tried by Andre Danière and Elizabeth Gilboy. Their approach attempts to keep numerical variables whenever there appears to be linearity, at least within ranges, and to repool groups whenever there does not appear any substantial nonlinearity or interaction effect. The method is feasible only where the number of factors is limited. The pooling both of groups and of ranges of "variables" makes it complicated. ${ }^{19}$ In practice, they found it useful to restrict the number of allowable interaction effects.

There are also studies going on in the sclection of test items to get the best prediction with a limited set of predictors. But the prediction equation in these analyses always seems to be multiple regression without any interaction effects. ${ }^{20}$ Group-screening methods have been suggested whereby a set of factors is lumped and tested and the individual components checked only if the group seems to have an effect. These procedures, however, require knowledge of the direction of each effect, and again assume no interaction effects. ${ }^{21}$ These groupscreening methods are largely used in experimental designs and quality control procedurcs. It is interesting, however, that they usually end up with two-level designs, and our suggested procedure of isolating one subgroup at a time has some similarity to this search for simplicity.

The approach suggested here bears a striking resemblance to Sewall Wright's path coefficients, and to procedures informally called "pattern analysis." The justification for it, however, comes not fron any complicated statistical theory, nor from some enticing title, but from a calculated belief that for a large range of problems, the real world is such that the proposed procedure will facilitate understanding it, and foster the development of better connections between theoretical constructs and the things we can measure.

One possible outcome, for those who want precise measurement and testing,

[^15]is the development of new constructs, as combinations of the measured "variables," which are then created immediately in new studies and used in the analysis. The family life cycle was partly theoretical, partly empirical in its development. Other such constructs may appear from our analysis, and then acquire theoretical interpretation.

## F. WHAT NEEDS TO BE DONE?

It may seem that the procedure proposed here is actually relatively simple. Each stage involves a simple search of groups defined as a subclass of any one classification and a selection of one with a maximum of a certain expression which is easily computed. It turns out, however, that the computer implications of this approach are dramatic. The approach, if it is to use the computer efficiently requires a large amount of immediate access storage which does not exist on many present-day computers. Our traditional procedures for multivariate analysis involve storing information in the computer in the form of a series of two-way tables, or cross-product moments. This throws away most of the interesting and potentially fruitful interconnectedness of survey data, and we only recapture part of it by multivariate processes which assume additivity. The implications of the proposed procedure are that we need to be able to keep track of all the relevant information about each individual in the computer as we proceed with the analysis.

Only an examination of the pedigree of the groups selected by the machine will tell whether they reveal things about the real world, or lead to intuitively meaningful theoretical constructs, which had not already come out of earlier "mullivariate" analyses of the same data.
It may prove necessary to add constraints to induce more symmetry, such as giving priority to seriatim splits on the same characteristic, since this might make the interpretation easier. Or we may want to introduce an arbitrary first split, say on sex, to see whether offsetting interactions previously hidden could be uncovered in this way.

Most statistical estimates carry with them procedures for estimating their sampling variability. Sampling stability with the proposed program would mean that using a different sample, one would end up with the same complex groups segregated. No simple quantitative measure of similarity seems possible, nor any way of deriving its sampling properties. The only practical solution would seem to be to try the program out on some properly designed halfsamples, taking account of the original sample stratification and controls, and to describe the extent of similarity of the pedigrecs of the groups so isolated. Since the program "trics" an almost unlimited number of things, no significance tests are appropriate, and in any case the concern is with discovering a limited number of "indexes" or complex constructs which will explain more than other possible sets.

It seems clear that the procedure takes care of most of the problems discussed earlier in this paper. It takes care of any number of explanatory factors, giving them all an equal chance to come in. It uses classifications, and indeed only those sets of subelasses which it actually proves important to distinguish. The results still depend on the detail with which the original data were coded.

Differential quality of the measures used remains a problem. Sample complexities are relatively unimportant since measures of importance in reducing predictive error are involved rather than tests of significance, and one can restrict the objective to predicting the sample rather than the population. Intercorrelations among the predictors are adequately handled, and logical priorities in causation can be.

Most important, however, the interaction effects which would otherwise be ignored, or specified in advance arbitrarily from among a large possible set, are allowed to appear if they are important.

There is theory built into this apparently empiristic process, partly in the selection of the explanatory characteristics introduced, but more so in the rules of the procedures. Where there is one factor of supreme theoretical interest, it can be held back and used to explain the differences remaining within the homogeneous groups developed by the program. This is a severe test both for the effect of this factor and for possible first-order interaction effects between it and any of the other factors used in defining the groups.

Finally, where it is desired to create an index of several related measures, such as attitudinal questions in the same general area, the program can be restricted to these factors and to five or ten groups, and will create a complex index with maximal predictive power.

# INPUT VARIABLES <br> TWO-STAGE WAGE-RATE ANALYSES 

(ISR Project 678, Deck 35)

| Variable | Column |
| :--- | :--- |
| Number | Number |

1
Physical condition--spending unit head
0 . SU head completely disabled

1. SU head severely disabled
2. SU head somewhat disabled, disabled but not limited, limitation NA
3. SU head reports no disability

2
9
E12. E13. Geographic mobility

1. lived in one state more than 100 miles from here
2. lived in two states
3. lived in three states
4. lived in four or more states
5. NA how many states lived in

0 . lived in one state less than 100 miles from her; head never worked

3
E4-E7. Education of the head of spending unit

1. grade school (1-8 years) or less
2. some high school (9-11 years); some high school plus noncollege training; grade school plus noncollege training
3. high school (12 years)
4. high school plus noncollege training, i.e., business college, trade school, etc.
5. college, no degree
6. college, bachelor's degree or no advanced degree mentioned
7. college, advanced degree
8. NA

0 . none

| Variable | Column |
| :--- | :--- |
| Number | Number |

413 Immigration of head or father
0 . spending unit head grew up in a foreign country
I. spending unit head grew up in the United States, father grew up in a foreign country
2. both spending unit head and father grew up in the United States
$514 \quad$ Occupation of spending unit head

1. professional, technical and kindred
2. managers and officials, nonself-employed
3. self-employed businessmen and artisans
4. clerical and kindred, sales workers
5. craftsmen, foremen, and kindred
6. operatives and kindred
7. laborers, farm and nonfarm, service workers
8. farmers and farm managers
9. government protective workers, members of the armed forces
0 . housewives, widows, students, rentier, never worked, occupation NA

615 Supervisory responsibility of spending unit head
0 . head is self-employed

1. head supervises others
2. head is neither self-employed nor supervisor

| Variable Number | Column <br> Number |  |
| :---: | :---: | :---: |
| 7 | 17 | Frequency of unemployment |
|  |  | 1. usual, seasonal, almost every year |
|  |  | 2. happens occasionally; every few years (a few times, 3 or more) |
|  |  | 3. short spells are usual, but not longer spells; unusual to be unemployed for more than a short period |
|  |  | 4. unusual to be unemployed, work is steady, seldom unemployed |
|  |  | 5. has never been unemployed |
|  |  | 6. entered labor force recently; was self-employed until recently (any other evidence of no experience) |
|  |  | 9. $\mathrm{DK}, \mathrm{NA}$ <br> Code 5 only if R says <br> he is never unemployed |
|  |  | 0. Inap., does not work for someone else |
| 8 | 19 | Rank in school of spending unit heads |
|  |  | 0 . head's grades above average |
|  |  | 1. head's grades average, $D K$, NA, and age less grades completed is 7 or less |
|  |  | 2. head's grades average, $D K, N A$, and age less grades completed is 8 or 9 |
|  |  | 3. head's grades below average and age less grades completed is 7 or less |
|  |  | 4. head's grades below average and age less grades completed is 8 or 9 |
|  |  | 5. head's grades not above average and age less grades completed is 10 or more |
|  |  | 6. head's grades not above average and had college training, or nonacademic training, has no education, retardation NA |

```
Variable Column
Number Number
    9 20 Religious preference and church attendance of
        spending unit head
            0. head is Catholic; attends two or three times
        a month or more, attendance NA
        1. head is Catholic; attends once a month or less
        2. head is Fundamentalist Protestant; attends two
        or three times a month or more, attendance NA
        3. head is Fundamentalist Protestant; attends
        once a month or less
        4. head is non-Fundamentalist Protestant, attends
        two or three times a month or more,
        attendance NA
    5. head is non-Fundamentalist Protestant, attends
        once a month or less
    6. head is non-Christian, religion NA
    10 Attitude toward hard work and need-achievement index
        hard work is equal to or more important than luck
        0. N/Ach score greater than . }3
        1. N/Ach score is between .15-.34
        2. N/Ach score is less than . }1
            luck is more important than hard work
            3. N/Ach score is greater than . }3
            4. N/Ach score is between .15-.34
            5. N/Ach score is less than . 14
            6. N/Ach score is NA
11 25 Ml. Race
    1. white
    2. Negro, other (Mexicans, Filipinos, Orientals,
    etc.)
12 Ag of head of spending unit
    1. Under }2
    2. 25-34
    3. 35-44
    4. 45-54
    5. 55-64
    6. 65-74
    7. 75 and over
```

Variable Column Number Number

13

14

15
32

Difference in the education of the spending unit head and his wife
0. no wife present

1. wife has two or more levels more education than head
2. wife has one level more education than head
3. wife has the same level of education as the head
4. wife has one level less education than the head
5. Wife has two or more levels less education than the head
6. education of wife NA

33 Urban-rural migration of spending unit head
head grew up on a farm:
0. lives in a rural area now

1. lives in a town $2,500-49,999$ now
2. lives in a city 50,000 or over now
head grew up in a small town or a city
3. lives in a rural area now
4. lives in a town or city 2,500 or over now
5. all other responses (NA where grew up, grew up in "other" or several places)

North-South migration of spending unit head
head did not grow up in the South
0 . moved into the South

1. does not live in the South now
head grew up in the South
2. is still in the South
3. moved out of the South
4. head grew up outside the United States
5. all other responses (NA where grew up, grew up in several regions)

| Variable | Columm <br> Number |
| :--- | :--- |

16. 35 Family composition

0 . single male head of $S U$, no children

1. single male head of $S U, 1$ or more children
2. single female head of SU, no children
3. single female head of $S U, 1$ or more children
4. married head of SU, no children
5. married head of SU, 1 child
6. married head of SU, 2 children
7. married head of $S U, 3$ or more children

17

18

19
44 Size of place

1. central cities of the 12 largest SMA's
2. cities 50,000 and over, exclusive of the central cities of the 12 largest PSU's
3. urban places $10,000-49,999$
4. urban places 2500-9999; urbanized areas not included in above codes
5. rural, near a city
6. rural, not near a city

Variable Column
Number Number

20
45
Difference in education of head and father
0. father had 1 or more levels of education more than the head

1. father had same education as the head
2. father had 1 level less education than the head
3. father had 2 levels less than the head

For fathers, levels of education are defined as:

1) 0-8 years, NA
2) $9-12$ years
3) some college or co1lege degree

For spending unit heads, levels of education are defined as:

1) 0-11 grades
2) 112 grades
3) college

21 52-54 Head's earning rate
(The quotient of head's total wage income divided by hours worked $x$ 100.)
xxx. Actual amount
-xx. Negative amount xx
998. Positive over the field amount $(N=4)$ * -98. Negative over the field amount $(N=16) * *$ 000. Head had no wage income $(N=451)$

2259 Sex of head of this adult unit

1. Male
2. Female
3. NA
*Self-employed businessmen and/or artisans, white **Primarily White, Farmers and also several self-employed businessmen and/or artisans
Variable Colunm
Number Number
23 64 Religious preference of head
4. Catholics
5. Fundamentalist Protestants
6. Non-Fundamentalist Protestants
7. non-Christians; not ascertained
24 65 Need-achievement score of head
8. under . 15
9. .15-. 34
10. . 35 and over
11. not ascertained
2566 Background of head
grew up in Deep South
12. on farm
13. in small town or large city
grew up outside Deep South in United States
14. on farm4. in small town or large city
15. grew up in foreign country
16. not ascertained
26 67-68 Weights
27 69-72 Interview number

## APPENDIX L

Listing of Sample Computer Input AID (2)
00000000011111111112222222222333333333344444444445555555555666666666677777777778 12345678901234567890123456789012345678901234567890123456789012345678901234567890 Input File follows:
15719 MTR 51 WAGE RATE - H AID-2 P. 678 - DECK 35. RUN3 2 W 2997 O27EXCLUD INOOOOOOOOOOOOOOOO21
309 026.02000.00500063025 O21HAGE RATE H TAP 27
001 M PHYS COND 003 M EDUCATION 008 F RANK IN SCHO 011 F RACE
4012 M AGE 022 F SEX 023 F RELIGIDN 024 F NEED/ACH
4025 F BACKGROUNO
 S4, 211,S6,13,54,11,54,311,12, $\mathrm{C} 4+1$
DATAFOLLOWS
$530031213022720400250111309710012302201003424000000000001252222314810271678-35$ $320030401112522412350011433943332700412003506700000000004111112322512481678-35$ $430332224622420450550011333762141602202402415100160001003122113342623731678-35$ $330030224022120406352311201410041220002002220000000000001222122142725021678-35$ $210031401012722401025111409610031300415004611000000000001212211144833711678-35$ $330031401121720455052011446670341552101401414000000000000141111143925821678-35$ $230030005012722406011011101510035230002103520000000000003252221262818241678-35$ $432034401112622412211011446676300601412033614621000910001141112232209821678-35$ $635036327312812450251011223222500602201053633026002420001141112234414891678-35$ $331043131012721405241512508110010221214043313025000450001242122232323541678-35$ 431033401012722401251512209110010303314043310023000530009212122232105271678 -35
$\stackrel{\bullet}{\bullet} \quad$ -
$036037422212522441221101334640310503202033413021003550001111122234806621678-35$ $634135203412812441211311123223201502102052613726141780433132122235313121678-35$ $423044332012421401311101101310001021002052510023001260001132212235843321678-35$ $533035504212712470321211433740501722301042524023001390001132212232618891678-35$ $534035401012622403354401508610001040011332510021001720000131112235811651678-35$ $421132441112722445354311657677301400302034606721090140674112212234814911678-35$ $836437326312122476291411557886511420103432436421093141781132112234802261678-35$ $735337305412522426210001224523410522105043434322052981031111112334815361678-35$ 635036421212522421350011223313210723312033413421002860000111112335815371678 -35 $736337442312522421220311546682201502102142616422023611171132122334620971678-35$ $625036542212621451210101334643311753304052414620002610001151222335808321678-35$ $523036421312521451210011532652111400305252415429001380001122112334832911678-35$ $534035404012622406210401301710021221001032220021001810001221212335819171678-35$ $735536402312612451110301333654211700305032413621162142001152221335312021678-35$ 536035001311512411110011433653111701301034415421003160001121111334804681678 - 35 $625036402312622411410001333650200702301033614021002860001141113334806121678-35$ E
IS719 MTR 51 WAGE RATE - H RESIDUALS P. 678 - DECK 35. RUN 4
$2 \quad 1 \quad 2997 \quad 028$
317 026.02000.00500063025 O28RESIDUALS 51
4002 M GEOG MOBILIT 003 M EDUCATION 004 M IMMIGRATION 005 F OCCUPATION 4006 F SUPR RESP 007 F FREQ OF UNEM 009 F REL X ATTEND 010 F HORK $X N / A C H$ 4011 F RACE 013 F H-W ED OIFF 014 F URB-RUR MTG 015 F N-S MIG
4016 F FAM COMP O17F INCOME COMM OLB M ABIL TO COMM OIG M SIZE OF PLAC
4020 F H-F ED DIFF


PRDGRAM ON TAPE 00002, ID= 00001

| MAP |  |  |  |
| :---: | :---: | :---: | :---: |
| DAYTIM | 00000* | WEFTAP | 00000* |
| CHEKIO | 00000* | RDSBIN | 00000* |
| SPRINT | 00000* | SCARDS | 00000* |
| CAS | 70163 | WRATIM | 70205 |
| - ERR | 74561* | . 0331 i | 74647* |
| SQRT | 76023* | .EXIT | 76102* |
| -RBIN | 76425* | :W8IN | 76661* |

00606 LOCS. CAN BE SAFELY USED IN EXPANDING PROG. (OCTALI
PROGRAM DN TAPE 00002, $10=00002$


| MAP |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| DAYTIM 00000* | SELRCU 00000* | SYSTEM 00000* |  |
| (MAIN) 10000 | WRATIA 67211 | -IOH 67263* |  |
| .O33II $73653 *$ | -PRINI 73670* | -PCOMT 73754* |  |
| ATLOC $74127 *$ | ZERO 74155* | SEQPGM 74211. |  |

## REWTAP 00000*

 SELRCD 00000* SPEEK 00000. 1 OH 70257 -PRINT 74664* PRINT 74664 RWT 77072 .PROGRAM ON TAPE $00002, I D=00003$

MAP
DAYTIM 00000 HEFTAH 00000 SCARDS 00000 . ERROR ERROR 00000 DFDP 71655 . - PCOMT 72313* -R8IN 72700* (ERAS) 77741
 SYSTEM 00000* MAIN) 10000 SUBT) 73752 READ 74750* ERRO 76205*
EFT 77117.
 ERROR 00000* RFORM 66460 DFDP 74412 PRSLT 75115* SEOPGM 76241* (PROG) 77133

HRSEIN 00000* SKIPG 00000* EDITPM 66460 DFMP 74412* PCOMT 76004 * PCOMT 76004* (IOB 76276


SOATA

```
NO. OF INPUT DATA OF VARIABLES 
NO. OF VARIABLES (OFDICTORS 27
HEIGHT VARIABIE
SPLIT ELIGIBILITY CRITERION .0200
SPLIT REDUCIBILITY CRITENION .0050
MAXIMIJM ALLDWARLE GRNUPS O
DEPENDENT VARIABLE IS 2I (WAGE RATE H)
VALUES OF OEPENDENT VARIARLE LARGER IHAN -.00000000E 00 ARE OMIITED.
```



```
QUTPUT OPYION I IS i. i.
OUTPUT OPTION 2 IS O.
MINIMUM SILE REQUIRED EXCLUOES OESERN.
INPUT DATA ARE ON CARD
RESIDUALS ARE REQUFSTEN AND OUTOUT WILL BE TAPE.
EXCLUDE DATA WHICH LIE INSIDE UF INTERVAL FROM O TUI O ON VARIARLE 2I
    WHICHLIE SIUE TF INTERVALFROM -O TO -O UNVAKIAHLE -O
```

|  |  |  |
| ---: | ---: | ---: |
| 0 | 1 | $C$ |
| 1. | .3 | 1 |
| 2 | 9 | 1 |
| 3 | 10 | 1 |
| 4 | 13 | 1 |
| 5 | 14 | 1 |
| 6 | 15 | 1 |
| 7 | 17 | 1 |
| 8 | 19 | 1 |
| 9 | 20 | 1 |
| 10 | 22 | 1 |
| 11 | 25 | 1 |
| 12 | 26 | 1 |
| 13 | 32 | 1 |
| 14 | 33 | 1 |
| 15 | 34 | 1 |
| 16 | 35 | 1 |
| 17 | 37 | 1 |
| 18 | 39 | 1 |
| 19 | 44 | 1 |
| 20 | 45 | 1 |
| 21 | $52-54$ | 1 |
| 22 | 59 | 1 |
| 23 | 64 | 1 |
| 24 | 65 | 1 |
| 25 | 66 | 1 |
| 26 | $67-68$ | 1 |
| 27 | $04-72$ | $C$ |

INPUT-DATA FORMAT AS FOLLOWS.
IC1,S1,I1,55,211,S2,3I1,S1,11,S1,2I1,S1,I1,S2,211,55,4I1,S1,I1,S1,I1,


## read data begins.

TIME IS NOW 12. 6. 56. 27.
data are all in.
TIME IS NOH 12. 9. 50. 46.

* PREDICTOR

LISIING.
VARIABLE ND.
UESCKIPTION MAXIMUM VALUE
$\mathrm{T}_{\mathrm{Y}}^{\mathrm{Y}} \boldsymbol{M} \boldsymbol{\mu} \mathrm{E}$
PHYS CIND
EOUCAIICN
RANK IN SCHO
RACE
AGE
SEX
RELIGION
NEED/ACH RACKGKOUND $\square$ $M$
1
3

3
8
11

22

23

| MAXIMUM VALUE |  |
| :---: | :---: |
| 3 |  |
|  | 7 |
| 9 |  |
|  | 2 |
|  | 7 |
|  | 2 |
|  | 4 |
|  | 4 |
|  | 6 |



A

* STATISTICS FOR TOIAL.

TOTAL ND. OF DATA READ 2997 NO. DF DATA DELETED 451
TOTAL NO. OF DATA USEO 2546
SUM DF WEIGHTS . 11676900 E 06
SUM OF Y $\quad .26937631 E 08$
SUM OF Y-SQUARE $\quad .86588781 E 10$
MEAN $Y$-2306.3163E 03
STANDARD DEV. $Y$.1446:3030E 03
TOTAL SUM OF SQUARES (TSA) .24445921E 10
$P A=4.889184 \mathrm{~L} 07$
$P A=1.222296 E 07$
TIME IS NOW 12. 9. 51. 9.


| 1 | 2197 | $.104888000^{\circ} 06$ | $.25037432 E O B$ | .82090334E 10 |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 349 | .118810002 95 | .19001990507 | .44984763E 09 |

- FOR. VARIABLE 11 (RACE ) $B S S=.66218176 E 08$

TRY ON VARIABLE 22 DVFR GRIUP 1 . RESULTS FOLLOW.

| CODE | $N$ | SUM DF WEIGHT | SUM OF | Y | SUM Y-SQUARE |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2162 | .97946000L 05 | . 24339005 E | 08 | . 81224824 ElO |
| 2 | 384 | .16823000605 | . 25986260 E | 01 | $.53640391 E 09$ |


| MEAN | STO. DEV. |  |
| :---: | :---: | :---: |
| $.24352155 E$ | 03 | $.14820919 E 03$ |
| $.15446864 E$ | 03 | $.89580077 E ~ U 2$ |

$.11419251 E \quad 09$
$.24446004 E \quad 10$
TRY ON VARIARLE 23 OVER GROUP 1 . RESULTS FOLLOW.

| CODE | N | SUM DF WEITHT | SUM DF | $Y$ | SUm Y -SQUARE |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 162 | . $77320000{ }^{\circ} \mathrm{O}$ | . 23630970 E | 07 | . 10157016 E | 10 |
| 3 | 965 | $.45422000 L 05$ | .10991533 E | 08 | . $37036491 E$ | 10 |
| 1 | 543 | .26428000 L 05 | .63744669E | 07 | $.19811031 E$ | 10 |
| 2 | 876 | . 37187000605 | $.72085340 E$ | 07 | . 14584594 E | 10 |

* FOR VARIABLE 23 ( RLLIGINN J BS S = . $74077503 E 08$
MEAN
$.30562558 E 03$
$.24198699 E 03$
$.24120126 E 03$
$.19384554 E 03$
H5S/TSS $=$

| STU. DEV. | H S S |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| .19482398E 03 |  |  |  |  |
|  | .46494592E 08 |  |  |  |
| .15159472E 03 |  |  |  |  |
|  | . $41212864 E 03$ |  |  |  |
| .12955397E 03 |  |  |  |  |
|  | . 74077503 E O8 |  |  |  |
| .12283758E 03 |  |  |  |  |
|  | . 24446273 E 10 |  |  |  |
| . 03030 |  |  |  |  | TRY DN VARIABLE 24 OVEY GRDUP 1 . RESULTS FDLLOW.



- for variable 241 NEED/ACH ) B S S $=.41804608 E 08$
MEAN
$.25737808 E 03$
$.22998855 E 03$
$.22943023 E 03$
$.1941544 S E 03$

STD. DEV. B $S \quad S$
. 14778575 F 03 .3615R20日E 08

- 3615R208E 08
.37999488 E 08
.41804608 E 08
$.24446248 E 10$


| GROUP | N | TOIAL WEIGHT | SUM IJF $Y$ | SUTA Y-SQUARE | $\Gamma \mathrm{S}$ | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 2262 | . LU317300E OS | . 22243408 E OB | . 65990852E 10 | . 18033399 E | 10 |
| 3 | 284 | . 15596000805 | .46937230E O7 | . 20398335E 10 | . $43942787 E$ | 04 |


| S | T E P | NO | 2 | Parent | Grinup |  | 2 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ＊ | FOR | VARIABLE | 1 | 1 PHYS COND | ） | 8 S | S | $=$ |
| ＊ | FOR | VARIABLE | 3 | （ ELUCATITN | 1 | B 5 | 5 | $=$ |
| ＊ | FOR | VARIABLE | 8 | （ R4NK IN SCHO | J | B S | S | $=$ |
| ＊ | FOR | VARIABLE | 11 | 1 RACE | 1 | E S | S | $=$ |
| ＊ | FOR | VARIABLE | 12 | 1 AuE | ） | R S | S | $=$ |
| ＊ | FOR | VARIABLE | 22 | 1 SeX | $) \quad B$ | B 5 | S | $=$ |
| ＊ | FOR | VARIABLE | 23 | 1 RELIGION | ） | $B$ S | S | $=$ |
| ＊ | FOR | VARIABLE | 24 | （ NeEb／ACH | ） | H S | S | $=$ |
| ＊ | FOR | VARIABLE | 25 | （ BACKGRDUNO | J． | B S | S | $=$ |


| ． 24916480 E | OR | BSS／TSS | ＝ | ． 01382 |
| :---: | :---: | :---: | :---: | :---: |
| ． 884227195 | 08 | BSS／TSS | $=$ | ． 04903 |
| ． $40076064 E$ | 08 | BSS／TSS | ＝ | ． 02223 |
| ． 49295232 E | 08 | BSS／TSS | $=$ | ． 02734 |
| ． $24694784 E$ | 08 | BSS／TSS | $=$ | ． 01647 |
| ． 96526976 E | 08 | BSS／TSS | ＝ | ． 05353 |
| ． 49552896 E | 08 | BSSITSS | ＝ | ． 02748 |
| ． 234858888 | 08 | BSS／ISS | $=$ | .01302 |
| ． 10887456 E | 09 | ESS／TSS | $=$ | ． 06037 |



CANDIDATE GROUPS ARE AS FOLLTWS．

| GROUP | N | TOIAL WEIGHT | SUM IJF Y | －SUn $\quad$ Y－SUUARE | 15 | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 284 | ．15596000E 05 | ． 46737230 E 07 | ．20，98335E 10 | ．43942787E | 09 |
| 4 | 1244 | ． $60244000 E 05$ | ． 14640504508 | ． 43135412 E 10 | $.10156043 E$ | 10 |
| 5 | 1018 | ．4 4724000 O | $.76034039 E 01$ | ．2025j397E 10 | ． 67895678 E | 04 |

 ＊FIJR VARIABLE 3 （ EUUCATIIN ，H S S＝
 －FOR VAKIALLE 11 i RaCE ，i S $S=.11041376 \mathrm{E}=\mathrm{CA}$ ＊FUR VARIABLE 12 （AUE ）＂$S$




．46つ37230E 07
76034U39E 01

SUM SUARE
，398335E 10 ．20くらら397E 10

7E 0 67895678 E



| CANOIDATE | GROUPS | ARE AS FOLLOWS. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| GROUP | $N$ | TOIAL WEIGHT | SIJM OF Y | SIJM Y-SOUARE | $T$ S | S |
| 3 | 284 | .13576000E 05 | . 46937230 E 07 | . 20598335E 10 | .43442787E | 07 |
| 5 | 1018 | . $42929000 E 05$ | . 76034039 E 07 | . 20255397 E 10 | . 67885678 E | 03 |
| 7 | 207 | . 71710000 E 04 | . 1528578OE 07 | . 27257008E 09 | . 53438915 E | 08 |
| 9 | 942 | .4.960000E 05 | . $12274607 E$ OY | $.40465205 E 10$ | . 81565139 F | 09 |


| . 213952006 | 07. | OSS/TSS | $=$ | . 00262 |
| :---: | :---: | :---: | :---: | :---: |
| . 33007552 E | 08 | HSS/TSS | = | . 04047 |
| . $20065760 E$ | CH | BSS/TSS | $=$ | . 02460 |
| . 727516795 | 07 | BSS/TSS | $=$ | .00892 |
| . 77309439 E | 07 | HSS/TSS | $=$ | .00948 |
| TEP= | 5 |  |  |  |
| . 98365434 E | 07 | RSS/TSS | $=$ | . 01206 |
| . 79752320 E | 07 | USS/TSS | $=$ | . 00980 |
| . $36856640 E$ | 07 | USS/TSS | $=$ | . 00452 |

DECOMPOSE GROUP 9 INTU GROUP 10 AND ll HY VARIARLE 3 iN S $\quad$ I E P 5.

| CODE | N | SUM OF hEiGHT |  | SUM DF | $Y$ | SUA Y-SCUARE | MEAN |  | STO. UEV. | $B$ S S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 2 | . 105000002 | 03 | . $24317000{ }^{\text {c }}$ | 05 | . 56865209507 | . 23159047 E | 03 | . 2287344902 |  |
| 1 |  |  |  |  |  |  |  |  |  | . 13324800606 |
|  | 234 | . 10956000t: | 05 | . 250009205 | 07 | . 75269352 E O9 | . 22519306 F | 03 | . 12895367563 |  |
|  |  |  |  |  |  |  |  |  |  | $\begin{aligned} & .27103704=08 \\ & .33007552=08 \end{aligned}$ |
| 2 | 241 | $.12039000=$ | 05 | . 303162605 | 07 | . 93592133 E 09 | . 2518104 E | 03 | . 11970354503 |  |
|  |  |  |  |  |  |  |  |  |  |  |
| 3 | 198 | . $975100000^{\circ}$ |  | . 27769790 E | 01 | . $95566031 E 09$ | . 28478914 E | 03 | . 130005 HGE $03^{\text {O }}$ | . 21651072608 |
|  |  |  |  |  |  |  |  |  |  |  |
| 4 | 118 | -57780000L | 04 | . 17627990 E | 07 | .63399083E 09 | . 305088095 | 03 |  |  |
|  |  |  |  |  |  |  |  |  |  | . 82230400 CO |
| 5 | 149 | .733100002 |  | .21837340 E | 07 | . 79256828 OC | . 297834875 | 03 | .139199485 03 |  |
|  |  |  |  |  |  |  |  |  |  | . 8156515 LL 09 |

CANDIDATE GROUPS ARE AS FOLLOWS.

| GROUP | $N$ | TOIAL WEIGHT | SUM OF Y | SUM Y-SQUARE | 15 | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 284 | . 13576000E O5 | .46937230E 07 | . 20593335E 10 | . $43942787 E$ | 09 |
| 5 | 1018 | . $42929000 E 05$ | . 76034039 E 07 | . 20255397E 10 | . 67885678 E | 04 |
| 7 | 207 | .97710000E 04 | .15285780E 07 | . 29257008E 09 | . 53438915 E | 08 |
| 10 | 477 | . 25100000E 05 | . 55560350 E 07 | . 16943013 E 10 | . 35795830 E | 09 |
| 16 | 465 | .22860000E OS | .67235719E 07 | $.24022194 E 10$ | . 42468573 E | 09 |







| RIABLE | 22 OVER |  | Jup 10 is |  |  |  |  |  | TEP = | 19 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FOR | VARIABLE | 23 | ( RELLIGION | ) | R | S | 5 | $=$ | . $51176160 \%$ | 07 | HSS/ros | = | . 01430 |
| FUR | VARIABLE | 24 | 1 NEED/ACH | 1 | R | 5 | S | $=$ | . 29413440 E | 07 | LiSS/Tis | $=$ | .00832 |
| FOR | VARIAILE | 25 | BACK(IRCIJN) | 1 | is | S | 5 | $=$ | -1'112430E | 07 | HSS/rss | $=$ | . 10422 |

FAILED TO SPLIT GROUP LU TRIED ON VARIABLE B, BUT BSS = . $68 G B 4640 E 01$

CANOIDATE GROUPS ARE AS FQLLINS.


CANDIDATE GROUPS ARE AS FOLLINS.

| GROUP | iv | TOIAL WCIGHI | SUM IIF Y | SUM Y-SDUARE | 1 S | $\leftrightarrows$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 207 | .91710000E 04 | . 1528'780E 07 | . 242:3008E O9 | .53438915 F | 08 |
| 12 | 477 | . 1c001000e 05 | . 25417880 O7 | . 5 d9C00502 04 | . 23021302 E | 07 |
| 14 | 97 | . 40590000504 | . 13505250 O 07 | .47244421E09 | . 80877499 F | 188 |
| 16 | 154 | . 74700000504 | . 19316540E 07 | . $58248462 E 09$ | .83386548E | 08 |
| 17 | 311 | . 13384000505 | .47919180 O | . 18197308 E 10 | . $3271100 Y E$ | UY |
| 20 | 162 | . 71260000 E 04 | . 30194150 E O7 | . 14865601 E 10 | . 30653592 E | 09 |



FAILED TO SPLIT GROUP 17 TRIED ON VARIABLE 12 . BUT PSS $=.41774880 E 07$

CANOIDATE GROUPS ARE AS FDLLOWS.

| GROUP | N |  | TOIAL WEIGHT |  |  |  |  | OF Y |  | SUM Y-SOU | ARE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 207 |  | . 97710000 E 04 |  |  |  | 15285 | 780E 01 |  | . 2Ч257008E | 09 |
| 12 | 477 |  | . Iv007000E 05 |  |  |  | 2541 | 480E 07 |  | -58900050E | 09 |
| 14 | 97 |  | .40580000E 04 |  |  |  | 1.3505 | 5250E 07 |  | . 47244421 E | 09 |
| 16 | 154 |  | . 74760000 O |  |  |  | 19316 | 540E 07 |  | . 58248862E | 09 |
| 20 | 162 |  | . 71260000 E O4 |  |  |  | 30194 | $150 E 07$ |  | . 14865601 E | 10 |
| FOR | VARIABLE | 1 | $($ Prys Conis | J | $\mu$ |  | S $=$ | . 28601600 E | 06 |  | BSS/TSS |
| FOR | VARIABLE | 3 | ( EUUCATION | 1 | B |  | $\mathrm{S}=$ | . 49199840 E | 07 |  | BSS/TSS |
| FOR | VARIABLE | 8 | 1 RANK IN SCHO | ) | B |  | $\mathrm{S}=$ | . $43687520 E$ | 07 |  | BSS/TSS |
| - For | VARIABLE | 11 | 1 Race | 1 | H |  | $S=$ | . $70482080 E$ | 07 |  | BSS/TSS |
| - for | VARIABLE | 12 | 1 AuE | 1 | B |  | $S=$ | . 11654080 E | 08 |  | HSS/TSS |
| - FOR | VARIABLE | 22 | ( Scx | 1 | ${ }^{6}$ |  | S $=$ | . 12848800E | 08 |  | HSS/TSS |
| - For | VARIABLE | 23 | ( KELIGIUN | 1 | B |  | $\mathrm{S}=$ | .77663840 E | 01 |  | RSS/TSS |
| FOR | VARIABLE | 24 | $1 \mathrm{NtED/ACH}$ | 1 | B |  | S | . 23200800 E | 07 |  | RSS/TSS |
| - FOR | VARIABLE | 25 | 1 BACKGROUND | 1 | A |  | S = | .67075840E | 07 |  | BSS/TSS $=$ |



CANDIDATE GROUPS ARE AS FOLLOWS.

| GROUP | N | TOIAL WEIGHT | SUM OF $Y$ | SUM Y-SOUARE | 1 S | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 207 | . 9/710000E 04 | . 15285780E 07 | . 29257008E 09 | . 53438915 E | 08 |
| 12 | 477 | .18007000 O | . $25417880 E 07$ | .58900050E O9 | . 23021302 E | 09 |
| 14 | 97 | . 40580000E 04 | . 13505250 E 07 | . $47244421 E 09$ | . 808774998 | 08 |
| 16 | 154 | . 74760000 E 04 | .19316540E 07 | . $58248862 E 09$ | . 83386548 E | 08 |
| 22 | 147. | .67309999E 04 | . 27.360940 E 07 | . $11906026 E 10$ | . 27840382 E | 09 |




FAILED TI SPLIT GROUP 24 TRIED ON VARIABLE 11 , BUT BSS $=.66031040 E 07$

CANDIDATE GROUPS ARE AS FOLLDWS.

| GROUP | N |  | TOTAL WEICHT |  |  |  |  |  |  |  | SUM Y-SQU | ARE |  | T S | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 207 |  | . 97710000 E 04 |  |  |  | 15285 | 80E | E 07 |  | . 29257008 E | 09 |  | . $53438915 E$ | 08 |
| 12 | 477 |  | . 18007000E 05 |  |  |  | 25417 | 80 E | E 07 |  | . 58900050 E | 09 |  | . 23021302 E | 09 |
| 14 | 97 |  | . 40580000 E 04 |  |  |  | 13505 | 50 E | E 07 |  | . $47244421 E$ | 09 |  | .80877499E | 08 |
| 16 | 154 |  | . 74760000 E 04 |  |  |  | 19316 | 40 E | E 07 |  | . 58248862 E | 09 |  | . 83386548 E | 08 |
| FOR | VARIABLE | 1 | 1 Prys Cond | J | R | 5 | S = |  | 60333119 E | 07 |  | BSS/TSS |  | . 02621 |  |
| FOR | VARIABLE | 3 | Eullcatinn | ) | A | S | S = |  | $20381520 E$ | 07 |  | BSS/TSS | $=$ | . 00885 |  |
| FOR | VARIABLE | 8 | ( RANK IN SCHO | 1 | A | 5 | $5=$ |  | $63719360 E$ | 07 |  | BSS/TSS |  | .02768 |  |
| FOR | VARIABLE | 11 | RACE | ) | 4 | 5 | S |  | 32285720E | 07 |  | BSS/TSS | $=$ | . 01402 |  |
| FOR | VAR IABLE | 12 | ( AuE | 1 | B | S | S |  | $38996560 E$ | 07 |  | BSSITSS |  | . 01694 |  |
| FOR | VARIABLE | 22 | 1 Slx | ) | B | 5 | S |  | 86842400 E | 07 |  | BSS/TSS | $=$ | .03772 |  |
| FOR | VARIABLE | 23 | 1 RELIGIDN | ) | A | S |  |  | 11372520 E | 07 |  | BSS/TSS | $=$ | . 00494 |  |
| FOR | VARIABLE | 24 | ( NEED/ACH | 1 | B | 5 | 5 |  | 24419160 E | 07 |  | BSS/TSS |  | . 01061 |  |
| FOR | VARIABLE | 25 | ( BACKGROUND | 1 | 8 | S | $\mathrm{S}=$ |  | $82242399 E$ | 06 |  | BSS/TSS |  | .00357 |  |

FAILED TO SPLIT GROUP $1 \angle$ TRIED UN VARIABLE 22 , BUT BSS $=.86842400 E 07$




| CAND 1 DA | TE GROUPS |  | E | 45 | FOLLOWS |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| group | N |  |  | total | WEIGHT |  |  |  |  |  |  |  |  | SUM | Y－S6u | are |  | 1 S | $s$ |
| 7 | 207 |  |  | ．47710 | OODOE 04 |  |  |  | 157 | 785 | 780 | E 01 |  | ． 2725 | 37008E | C9 |  | ． 5343891 be | O\％ |
| －FOR | VARIABLE | 1 | 1 | Piry | ？ | 1 | P | 5 | S | $=$ |  | $14900700[$ | 07 |  |  | 9SS／ISS | $=$ | ． 02789 |  |
| －fijr | VAKIAGLE | 3 | （ | Eujuca | ATIOA | 1 | ค | S | S | $=$ |  | 49192800 E | 07 |  |  | ESS／TSS | $=$ | ． 09205 |  |
| －For | VARIABLE | 3 | 1 | KıNK | IV SCHO | ） | $\because$ | S | S | $=$ |  | 40700720 E | 07 |  |  | 13SS／TSS | $=$ | ． 07616 |  |
| －FOR | VARIABLF． |  | 1 | Ract |  | ， | R | S | S | $=$ |  | 90757400E |  |  |  | MSS／TSS | $=$ | ． 01698 |  |
| －For | VIRIAHLE | 12 | 1 | AUE |  | 1 | B | S | S | $=$ |  | H8510749E | 06 |  |  | 95S／ISS | $=$ | ． 01656 |  |
| VARIABL | 22 ПVIR |  | duu |  | 1151 | CON | T＾ | V |  |  | S T | $E \mathrm{P}=$ |  |  |  |  |  |  |  |
| －FOR | VARJARLE | 23 | 1 | ふilig | ginw | 1 | A | S | S | $=$ |  | 1765\％240E | 07 |  |  | BSS／TSS | $=$ | ． 07046 |  |
| －For | VARIABLE |  | 1 | NLEC）／ | Ach | 1 | $\cup$ | 3 | S | $=$ |  | 17400910E | 07 |  |  | BuS／ISS | $=$ | ． 0364 ？ |  |
| －fijr | VARIABLE | $\angle 5$ | 1 | backi | ，${ }^{\text {NOUN0 }}$ | 1 | H | S | S | $=$ |  | 327862005 | 06 |  |  | MSS：「ら | $=$ | ．0＾604 |  |

CANDIDATE GROUPS ARE AS FOLLOWS.
GROUP NOTAL WEIGHT

SUM DF Y
SUM Y-SQUARE
$T \quad S$
thaì is all. no more gruups are available. final stef no. is 13 no. of groups alre 25 .

* this is the end of zind core.

T\&ME IS NON 12. 10: 44. 7.

OEPENDENT VARIABLE 21 ( hage Rate H)

* total group

| $N=$ | 2546 |
| ---: | ---: | ---: |
| TOTAL WT SUM $=$ | 116769 |

## 

WEIGHTED BY VARIABLE 26
MEAN $=.23069163 E 03$ $.14469030 E 03$
SUM $Y=.26937631 E O B$ SUM Y SQ. $=.86588781 E 10$

GROUP NO
VALUES
$\mathrm{N}=$
2 SPLIT
PREDICTOR
2262
103173 PCT OF TOTAM =

- GRDUP 88.4

GROUP NO
3 SPLIT
PREDICROR
284
13596 HEIGHT SUM =
11.6

- GROUP NO.

VALUES NO. $\mathrm{N}=$

4 SPLIT
1244
60244
WEIGHT SUM $=$
PCT DF TOTAL =
51.6


GKOUP DEVIATION $=-.15093479 E 02$ TSS(I) $=.18033398 \mathrm{E} 10$ HTD. MEAN SQ. $=.47957454 \mathrm{E} 10 \quad$ (TSSII)/TSSITI) $=.73768536 \mathrm{E} 00$

| FROM | GRDUP | 1 | ON | VARIABLE | 3 | I Ecucation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 NCLUD | EED ARE |  | 6 | 7 |  |  | INCLUDED ARE $\quad{ }^{6} \quad \begin{aligned} & 7 \\ & \text { MEAN }\end{aligned}=.34522822 E 03$

STD. DEV. = . 17977870 E 03
WTD. MEAN SQ. $=.16204056 E 10$
$\begin{aligned} \text { GROUP DEVIATION } & =.11453660 E 03 \\ \text { TSSII) } & =.43942789 E 09\end{aligned}$
(TSS(I)/TSS(T)) $=.17975509 E 00$
FROM GROUP 2 DN .VARIABLE 25 IBACKGRDUND J
INCLUDED ARE 4 ON 5
MEAN $=.24302012 E 03$
STD. DEV. $=.12983906 \mathrm{E} 03$
WTD. MEAN SO. $=.355 .79370 E 10$
TSS(1) $=.12328487 E 02$
TSS(I) $=.10156043 E 10$
$(T S S(I) / T S S I T)=.41544937 E 00$

SUM $Y$ SQ $Y=.14640504 E 08$ SUM Y SO. $=.45735412 \mathrm{E} 10$

- GROUP NO. 5 SPLII FROM GRUUP 2 ON VARIABLE 25 (BACKGROUNO)

VALUES OF
NEHT $\mathrm{N}=$
PCT OF TDTAL =

- GROUP NO

VALUES OF
WEIGHT SUM $=$

PREDICTOR
1018
42929
36. 8

1NCLUDED $A R E \quad 1 \quad 2 \quad 3 \quad 3$
STD. DEV. $=.12575165 \mathrm{E} 03$
WTO. MEAN SQ. $=.13466829 E 10$
FROM GROUP 4 UN VARIABLE 22 ISEX
$\begin{array}{llll}6 \text { SPLIT } & \text { FROM GROUP } & 4 & \text { ON } \\ \text { PREDICTOR } & \text { INCLUDED ARE } & 1\end{array}$
1037 MEAN $=.25978099 E 03$
STD. DEV. $=.13164685 E 03$
WTD. MEAN SQ. $=.34062291810$

GROUP DEVIATION $=-.53575834 E 02$
TSS(I) $=.67885677 \mathrm{E} 09$
(TSSI!)/TSS(T) $=.27769735 \mathrm{E} 00$

GROUP DEVIATION $=.29089361 E 02$
TSS(1) $=.87474220 \mathrm{E} 09$
(TSS(I)/TSS(J)) $=.35782747500$

- GROUP INO. 7 SPLIT FROM GROUP 4 ON VARIABLE 22 (SEX VALUES OF PREDICTOR INCLUDED ARE ${ }^{4}$ ON ** THIS GROUP IS RETAINED AS DVE DF FINALS.
$N=$
HEIGHT SUM $=$


## 207 9771

8.4
$\begin{aligned} \text { MEAN } & =.15644028 E 03 \\ \text { STO. DEV. } & =.73953599 E 02\end{aligned}$
WTD. MEAN SQ $=.73953599 E 02$

GRDUP DEVIATION $=-.74251348 E \quad 02$
TSS(I) $=.5343891 \mathrm{AE} 08$ (TSS(I)/TSS(T)) $=.21860055 \mathrm{E}-01$

- GROUP NO. 8 SPLIT FROM GROUP 6 ON VARIABLE 12 (AGE VALUES OF PREDICTOR INCLUDED ARE I
THIS GROUP IS RETAINED AS ONE OF FINALS.
*. THIS GROUP IS RETAINED AS ONE OF FINALS. $\quad$ ME $=$ MEAN $=.18442699 E 03$
HEIGHT SUM $=$ STD. UEV $=4513$. $4842699 E 03$
PCT OF TOTAL =
- Group no.

VALUES OF
$N=$

$$
\text { WTD. MEAN SQ. }=.15350208 E 09
$$

GROUP DEVIATIUN $=-.46264641 E 02$
TSS(I) $=.30955248 \mathrm{EH}$
ITSS(I)/TSS(T)) $=.12662745 \mathrm{E}-\mathrm{CL}$

SUM Y $=.76034039 E 07$

SUAA $Y=.03231900 E 06$ SUM Y SU. $=.18445733 E 09$

SUM Y SO. $=.20255397 E 10$

SUM $Y=.13111926 E 08$ SUM Y SO. $=.42809713 E 10$

```
            Sum Y = .152B5780E 07
``` SUM Y SO. \(=.29257008 \mathrm{E} 09\)
- GROUP NO. 10 SPLIT FROM GROUP Y DN VARIABLE 3 (EDUCATION )

YALUES. OF PREDICTIR. INCLUDED ARE 0 O 1122

GROUP DEVIATION \(=.98293647 E 0\)
TSS(I) \(=.35795830 E 09\)
(TSS(I)/TSS(T)) \(=.14642864 E 00\)

\(\begin{aligned} \text { GROUP DEVIATION } & =.63427877 E 02 \\ \text { TSSII } & =.42468574 \mathrm{E} 09 \\ (T S S(I) / T S S(T) & =.17372458 \mathrm{E} 00\end{aligned}\)
- GROUP NO. 12 SPLIT FROM GROUP 5 ON VARIABLE 3 (EDUCATION )

VALUES OF PREDICTOR INCLUDED ARE 0 I
*. THIS GROUP IS RETAINED AS ONE OF FINALS.
\(\mathrm{N}=\mathrm{H}=47 \mathrm{MEAN}=.14115555 \mathrm{E} 03\)
PCT OF TOTAL \(=\) STD. DEV. \(=.11306919 E 03\)

GROUP DEVIATION \(=-.89536079 E 02\)
TSS(I) \(=.23021302 \mathrm{E} 09\)
(TSS(I)/TSS(T)) \(=.44172364 \mathrm{E}-01\)
* GROUP NO. 13 SPLIY FROH GRDUP 5 ON VARIABLE 3 (EUUCATION )

VALUES O
PREDICTOM INCLUDED ARE \(2 \quad 3\)
MEAN \(=.20309831503\)
WEIGHT SUM \(=\quad 24922 \quad\) STD. DEV. \(=.12803320 E 03\)
PCT OF TOTAL =
21.3

WTD. MEAN SQ. \(=.10280056 E 10\)

GROUP DEVIATIGN \(=-.27593322 E 02\)
TSS(I) \(=.40853389 \mathrm{E} 09\)
\((T S S(I) / T S S(T))=.16711740 E 00\)
- GROUP NO. 14 SPLIT FROM GROUP 3 ON VARIABLE 12 (AGE )

VALUES OF PREDICTIR INCLUDED ARE 1
** THIS GROUP IS RETAINED AS ONE OF FINALS.
WEIGHT SUM \(=\quad 97 \quad\) MEAN \(=.2 .8993666 E 0\)
PCT OF TOM \(=44658\) STD. DEV. \(=.13176926 E 03\)
* GROUP NO. 15 SPLIT FROM GRDUP 3 JN VARIABLE 12 IAGF
\(\begin{array}{lclllllclll}\text { GROUP } & \text { NO. } & 15 & \text { SPLIT } & \text { FROM GRDUP } & 3 & \text { DN } & \text { VARIABLE } & 12 & \text { IACF } \\ \text { VALUES } & \text { OF } & \text { PREDICTOR } & \text { INCLUDED ARE } & 3 & 4 & 5 & 6 & 7\end{array}\)

PCT OF TOTAL \(=7.7\) WTD. MEAN SQ. \(=.12505004510\)
\(\begin{array}{llllllll} & \text { GROUP NO. } 16 \text { SPLIT FROM GRIOUP } 11 & \text { ON VARIABLE } 12 \text { IAGE } \\ \text { VALUES OF PREDICTOK INCLUOED AKE } & 2 & \end{array}\)


WEIGHT SUM \(=\quad 747 \mathrm{~N}_{\mathrm{O}} \quad\) STO. OEV. \(=.10561202 \mathrm{E}\) O3
PCT OF TUTAL \(=6.4\) WTD. NEAN SO. \(=.49910208 E 09\)
GROUP DEVIATION \(=.59245035 \mathrm{E} 02\)
TSS(I) \(=.80877504 E 08\) (TSS(I)/TSS(T)) \(=.33084252 \mathrm{E}-01\)
\begin{tabular}{l} 
- GRDUP NO. 17 SPLIT FROM GROUP IL ON VARIABLE \\
VALUES OF PREDICTOR IACLUDED ARE \\
\hline
\end{tabular}
** THIS GROUP IS R[IAINEIT AS DNE DF FINALS. \(\begin{gathered}\text { N } \\ \text { N } \\ \text { HEAN }\end{gathered}\)
HEIGHT SUM \(=\quad 15344\) SII. DEV. \(=.14581840 \mathrm{E}\) O3
PCT OF TOTAL \(=13.2\)
WTD. MEAN SIJ. \(=.14320208\) E 10
\(\begin{aligned} \text { LRUUP DEVIATION } & =.27689054 F 02 \\ \text { TSSII } & =.83386543 E 08 \\ \text { (TSSISTISSITI) } & =.34110616 E-01\end{aligned}\)

GROUP DEVIATIDN \(=.14335156 E\) OJ
TSS(I) \(=.33688910\) E 09 (TSS(1)/TSS(T)) \(=.13780994[00\)
* GROUP NO. 18 SPLIP FROM GROUP 13 TN VARIARLE 22 (SEX

VALUES OF MREDICTIJR INCLUDEO ALE
THIS GROUP IS RETAINEO AS ONE OF FINALS.


SUM \(Y=.13505250\) E 07 SUM Y SQ. \(=.47244421 E 09\)

SUM \(Y=.33431980 E \quad 07\) SUM Y SQ. \(=.15873895 \mathrm{E}\) 10

SIJIA \(Y=.19316540 E 07\) SUAT Y SSA. \(=.58248862 \mathrm{E} 09\)

GKIJUP DEVIATION \(=.80795500 E\) O2
TSS(1) \(=.32711004507\)
\((T S S(1) / T S S(T))=.13380968 \mathrm{E} 00\)

SUM \(Y=.55560350 E 07\) SUM Y SO. = . \(16943013 E 10\)

SUM \(Y=.67235719 \mathrm{E} \quad 07\) SUM Y SU. \(=.24022194\) E 10

SUM \(Y=.25417880 E 07\) SUH Y SQ. \(=.58900050 \mathrm{E} 09\)

SIJM Y \(=.50616159 E 07\) SUH Y SO. \(=.14365395 \mathrm{E} 10\) SUM Y SO. \(=.18197308 \mathrm{E} 10\)


residuals are ortaineo.
TIME IS NOW 12. 12. 32. 14.
- results are on tape.

TIME IS NOW 12. 12. 32. 20.
```

NO. OF INPUT DATA 2997
NO. OF VARIABLES
NO. OF PREDICTORS
WEIGHT VARIABLE NO.
SPLIT ELIGIBILITY CRITERION .0200
SPLIT REDUCIBILITY CRITEKION .0050
MAXIMUM ALLOWABLE GROUPS 63
DEPENDENT VARIABLE IS 28 (RESIOUALS 5II
VALUES OF DEPENDENT VARIABLE LARGER THAN -.00000000E 00 ARE OMITTED.

```

```

OUTPUT OPTION 1 IS 1.
MINIMUM SILE REQUIRED 25
INPUT DATA ARE IN TAPE
RESIDUALS ARE NOT REQUESTED AND OUTPUT WILL BE NONE .
NO FILTERS.
REAO DATA BEGINS.

```
TIME IS NOW 12. 12. 36. 17.
OATA ARE ALL IN.

TIME IS NOW 12. 14. 21. 13.



TRY ON VARIARLE 2 DVER GRDUP 1 . RESULTS FOLLOW.


TRY ON VARIARLE 3 DVER GRTUP 1 . RESULTS FQLLOW.
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline code & N & SUM OF WEIG & , HT & SUM OF & \(Y\) & \multicolumn{2}{|l|}{SUM Y~SQUARE} \\
\hline 0 & 26 & . 79700000 L & 03 & -. 37707000 E & 05 & . 61453310 E & 07 \\
\hline 1 & 735 & . 30490000E & 05 & -. 163341005 & 06 & .41991119E & 09 \\
\hline 2 & 558 & . 26386000 L & 05 & -. 31696000 E & 05 & . 36308498 E & 09 \\
\hline 3 & 408 & . \(19431000{ }^{\circ}\) & 05 & -. 10491500 E & 06 & . 27635619 E & 09 \\
\hline 4 & 236 & . 11613000 L & 05 & . 13853600 E & 06 & \(\therefore 17570759 E\) & 09 \\
\hline 5 & 299 & .144560000 & 05 & . 20589300 E & 06 & . 23672578 E & 09 \\
\hline 6 & 212 & . 10165000 E & 05 & -. 11011100 E & 06 & . 26055941 E & 09 \\
\hline 7 & 72 & . 343100005 & 04 & . 11248800 E & 06 & . 11542463 E & \(09^{\circ}\) \\
\hline
\end{tabular}
* FOR VARIABLE 3 (EUUCATIAN ) \&SS = . \(35102028 E 07\)
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline MEAN & & \multicolumn{2}{|l|}{STO. DEV.} & \multicolumn{4}{|l|}{\multirow[t]{2}{*}{B S S}} \\
\hline -. \(47311167 E\) & 02 & .13974535 E & 02 & & & & \\
\hline & & & & \multicolumn{4}{|l|}{. 180217378} \\
\hline \multirow[t]{2}{*}{-. 53571990 E} & \multirow[t]{2}{*}{01} & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{.11723223E 03}} & & & & \\
\hline & & & & \multicolumn{4}{|l|}{. 1808050SE 07} \\
\hline \multirow[t]{2}{*}{-. 12012431 E} & \multirow[t]{2}{*}{01} & \multirow[t]{2}{*}{. 11729907 E} & \multirow[t]{2}{*}{03} & & & & \\
\hline & & & & \multicolumn{4}{|l|}{. 19286296E} \\
\hline \multirow[t]{2}{*}{-. 53995618 E} & \multirow[t]{2}{*}{01} & \multirow[t]{2}{*}{\(.11913557 E\)} & \multirow[t]{2}{*}{03} & \multicolumn{4}{|l|}{\multirow[t]{2}{*}{\[
.45102028 \mathrm{E} \quad 07
\]}} \\
\hline & & & & & & & \\
\hline \multirow[t]{2}{*}{. 119293895} & \multirow[t]{2}{*}{02} & \multirow[t]{2}{*}{. 12242524 E} & \multirow[t]{2}{*}{03} & \multicolumn{4}{|l|}{\multirow[t]{3}{*}{. \(19924657 E 07\)}} \\
\hline & & & & & & & \\
\hline \multirow[t]{2}{*}{. 14242736 E} & \multirow[t]{2}{*}{02} & \multirow[t]{2}{*}{. \(12717213 E\)} & \multirow[t]{2}{*}{03} & & & & \\
\hline & & & & \multicolumn{3}{|l|}{\multirow[t]{2}{*}{. 14289819 E}} & 03 \\
\hline \multirow[t]{2}{*}{-. 10832366 E} & \multirow[t]{2}{*}{02} & \multirow[t]{2}{*}{. \(15973621 E\)} & \multirow[t]{2}{*}{03} & & & & \\
\hline & & & & \multicolumn{4}{|l|}{\multirow[t]{2}{*}{. 37813848 E 07}} \\
\hline . 32785193 E & 02 & . 18046279E & 03 & & & & \\
\hline & & & & & 539 & 44 E & 10 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline CODE & N & SUM OF WEIGHT & SUM OF & \(\gamma\) & SUM Y-SOUARE \\
\hline 0 & 113 & . 55620000 L 04 & -. 36174000 E & 05 & . 12004495E 09 \\
\hline 1 & 367 & . 17867000505 & . 15258700 E & 06 & .31361849 E 09 \\
\hline 2 & 2066 & . 93340000505 & -. 10726800 E & 06 & . \(14202484 E 10\) \\
\hline
\end{tabular}
MEAN
-.65037750 E
01
.85401578 E
01
\(-.11492179 E\)
STD. DEV. B S S
.25301973506 \(.70098596 E G 6\) .18539112 E 10

\section*{TRY DN VAKIABLE 5 OVER GRDUP 1 . RESULTS FDLLOW.}
\begin{tabular}{ccccccc} 
COOE & N & SUM OF WEIGHT & SUM OF Y & SUM Y~SQUAKE \\
1 & 274 & \(.13469000 E O 5\) & \(.49634800 E O\) & \(.321302 B 8 E O 9\) \\
2 & 136 & \(.66810000 E 04\) & \(.23380200 E 06\) & \(.16224433 E 09\) \\
5 & 409 & \(.20177000 E 05\) & \(.64484300 E 06\) & \(.22747824 E 09\)
\end{tabular}
MEAN
\(.36851139 E \quad 02\)
\(.34995060 E \quad 02\)
\(.31959310 E \quad 02\)

B S S
\(.20588112 E 08\)
.31837146 E O8
.71285823 E 08

TRY ON VARIABLE 6 DVER GKOUP 1 . RESULTS FOLLOW.

\begin{tabular}{cc} 
MEAN & STD. DEV. \\
\(.30366630 E ~ O 2\) & \(.127156 I O E ~ O 3\) \\
\(-.31908315 E ~ O 1\) & \(.10400092 E 03\) \\
\(-.30844590 E ~ O 2\) & \(.18197270 E 03\)
\end{tabular}

3 5
- 313231ع7E OH \(.20690552 E 08\) .18539135510

A \(S \quad S\)
.13399565 E 07
. FOR VARIABLE 7 (FKEQ OF UNEM) BS \(S=.41763153 E 08\)
BSS/TSS \(=.01690\)
TRY ON VARIABLE 7 DVER GROUP 1 . RESULTS ḞOLLOW.
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline CODE & N & \multicolumn{2}{|l|}{SUM OF WEIGHT} & SUM OF & \(Y\) & \multicolumn{2}{|l|}{SUM Y-SOUARE} \\
\hline 3 & 44 & . 20550000 E & 04 & . 52172000 E & 05 & . 24341138 E & 08 \\
\hline 5 & 540 & \(.25747000 c\) & 05 & . 38930000 E & 06 & . 33849911 E & 09 \\
\hline 2 & 49 & . \(22310000 t\) & 04 & . 29698000 E & 05 & . 20891052 E & 08 \\
\hline 4 & 913 & . \(43944000 E\) & 05 & . 44658400 E & 06 & . 46345082 E & 09 \\
\hline 9 & 217 & . 98360000E & 04 & . 82740000 E & 05 & . 15538102 E & 09 \\
\hline 1 & 160 & . 63950000 E & 04 & -. 10692600 E & 06 & . 93087398 E & OB \\
\hline 0 & 578 & . 24347000 E & 05 & -. 76388900 E & 06 & . 74235622 F & 09 \\
\hline 6 & 45 & . 22140000 E & 04 & -. 12053400 E & 06 & . 15908064 E & 08 \\
\hline
\end{tabular}

TRY ON VARIARLE 9 OVER GRQUP 1 - RESULTS FOLLON.
\begin{tabular}{cccccc} 
CODE & N & SUM OF WEIGHT & SUM OF & Y \\
6 & 162 & \(.77320000 E\) & 04 & \(.31032700 E\) & 06 \\
1 & 112 & \(.54575999 E\) & 04 & \(.11789000 E\) & 06 \\
4 & 455 & \(.21122000 E 05\) & \(.12677300 E\) & 06 \\
5 & 510 & \(.24300000 L\) & 05 & \(-.19152000 E\) & 05 \\
0 & 431 & \(.20970000 E\) & 05 & \(-.10262500 E\) & 06
\end{tabular}
\begin{tabular}{cc} 
SUM Y-SDUARE & MEAN \\
\(.24378307 E\) O9 & \(.40135411 E ~ O 2\) \\
\(.10976898 E 09\) & \(.21599487 E 02\) \\
\(.41072353 E ~ 09\) & \(.60019410 E 01\) \\
\(.40071176 E 09\) & \(-.78 B 14814 E 00\) \\
\(.24647000 E 09\) & \(-.48438960 E ~ 01\)
\end{tabular}

STD. DEV.
\(.17296895 E 03\)
\(.14016076 E 03\)
.13931716 E 03
.12841174803
\(.10830285[03\)
\(13286310 E O 8\)
\(.15596997 E 08\)
\(.12589499 E 08\)
.96679214507
\begin{tabular}{llllllll}
2 & 499 & .208350001 & 05 & \(-.16926600 E 06\) & \(.27341738 E 09\) \\
3 & 377 & \(.16352000 E 05\) & \(-.25480200 E 06\) & \(.16904052 E 09\)
\end{tabular}
- For variable 9 (kEL X ATtENU) bS \(=.15596997 E 08\)
\begin{tabular}{llll}
\(-.81241179 F\) & 01 & \(.11426716 E\) & 03 \\
\(-.15582314 E 02\) & \(.10047286 E\) & 03
\end{tabular}

TRY ON VARIABLE 10 GVER GROUP 1 . RESULTS FOLLOW.
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline CODE & N & \multicolumn{2}{|l|}{SUM OF WEIGHT} & SUM DF & \(Y\) & \multicolumn{2}{|l|}{SUM Y-SCUARE} \\
\hline 0 & 624 & . 30109000E & 05 & . 44443600 E & 06 & . 51722389E & 09 \\
\hline 3 & 120 & . 52770000 i & 04 & . 25273000 E & 05 & .65230231E & 08 \\
\hline 1 & 913 & . 426310002 & 05 & . 65183999 E & 05 & . \(69837156 E\) & 09 \\
\hline 4 & 227 & . \(10017000 \dot{E}\) & 05 & -. \(30291000 E\) & 05 & . 16640484 E & 09 \\
\hline 6 & 90 & . \(40420000 \dot{\text { c }}\) & 04 & -. 45915000 E & 05 & . \(10507942 E\) & 09 \\
\hline 5 & 138 & . 54929999 E & 04 & -. 93246999 E & 05 & .68130314E & 08 \\
\hline 2 & 434 & . \(19200000 i\) & 05 & -. 35629500 E & 06 & . 23347454 E & 09 \\
\hline
\end{tabular}
\(M E A N\)
\(.14760702 E \quad 02\)
\(.47892741 E \quad 01\)
\(.15290282 E 01\)
\(-.30239592 E 01\)
\(-.11359475 E \quad 02\)
\(-.16975605 E \quad 02\)
\(-.18557031 E \quad 02\)
\begin{tabular}{cc} 
STD. UEV. \\
\(.13023247 E\) & 03 \\
\(.11107789 E\) & 03 \\
\(.12798218 E\) & 03 \\
\(.12885301 E\) & 03 \\
\(.16083485 E\) & 03 \\
\(.11006791 E\) & 03 \\
\(.10870036 E\) & 03
\end{tabular}
\(B S S\)
.87460156507
.88405690 E 07 \(.10799306 E O B\) \(.11434421 E 08\) .10468305 E 08 .79797971 E 07 \(.18539140 E 10\)

TRY ON VARIABLE 11 OVER GROUP 1 . RESULTS FOLLOW.
\begin{tabular}{|c|c|c|c|c|c|}
\hline CODE & N & SUM OF WEIGHT & SUM OF & \(Y\) & SUM Y-SQUARE \\
\hline 1 & 2197 & . 10488800 O 06 & . \(42918500 E\) & 06 & . 17250888E 10 \\
\hline 2 & 349 & \(.11881000 E\) OS & -. 42004000 E & 06 & \(.12882250 E 09\) \\
\hline
\end{tabular}
MEAN
.40918408 EL
\(-.35353926 E 02\)

STD. DEV.
\(.12818041 E 03\)
\(.97943003 E 02\)
* FOR VARIABLE 11 ( RACE , BSS \(=.16605503 E 0 B\)
BSS/TSS =
.00896
TRY DN VARIARLE 13 OVER GRDUP 1 . RESULTS FOLLOW.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline CODE & \(N\) & \multicolumn{2}{|l|}{SUM TF WEIGHT} & Sum OF & \(Y\) & \multicolumn{2}{|l|}{Y-SGUARE} & \multirow[t]{2}{*}{\[
\begin{gathered}
\text { MEAN } \\
.27074954 E
\end{gathered}
\]} & \multirow[b]{2}{*}{02} & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{\[
\begin{gathered}
\text { STD. DEV. } \\
.16168231 E 03
\end{gathered}
\]}} \\
\hline 6 & 13 & .54700000 c & 03 & . 14810000 E & 05 & . 14700200 E & 08 & & & & \\
\hline 1 & 212 & . 99620000 E & 04 & .26039300 E & 06 & . 18686364 E & 09 & . \(26138626 E\) & 02 & . 13444112 E & 03 \\
\hline 2 & 341 & . 154100002 & 05 & . \(22254400 E\) & 06 & . 24107806 F & 09 & . \(14441531 E\) & 02 & . 12424050 E & 03 \\
\hline 4 & 319 & . 154470005 & 05 & . 14432000 E & 06 & . 27967078 E & 09 & . 93429144 E & 01 & . 13423075 E & 03 \\
\hline 3 & 726 & . \(327050000^{\circ}\) & 05 & . 129736008 & 06 & . 570172995 & 09 & . 39668552 E & 01 & . 13427438 E & 03 \\
\hline 5 & 285 & . 137150000 & 05 & -. \(68479999 E\) & 05 & . 26662249 E & 09 & -. 50295296E & 01 & . 13933741 E & 03 \\
\hline 0 & 650 & . 28983000 E & 05 & -. 69367800 E & 06 & . 27480699 E & 09 & -. 23933961 E & 02 & . 44386581 E & 02 \\
\hline
\end{tabular}
- FOR VARIABLE 13 (H-WED DIFF) A S \(S=.22228613 E 08\)

RSS/TSS \(=\)
.01199
TRY ON VARIABLE 14 GVER GROUP 1 - RESIJLTS FOLLOW.
\begin{tabular}{ccccccc} 
COOE & N & SUM OFWEIGHT & SUM OF & Y & SUM Y-SOUARE \\
2 & 134 & \(.63559999 E 04\) & \(.32991900 E 06\) & \(.95259336 E O B\) \\
3 & 444 & \(.2043 R 000205\) & \(.19536900 E 06\) & \(.36417398 E 09\)
\end{tabular}
\begin{tabular}{cc} 
MEAN & SYO. DEV. \\
.51906702 E & 02 \\
.95591055 E & 01
\end{tabular}
\(B \quad 5\)
.18054209208
.18539105 E 10

B \(S\)
.40053998 E 06
.7872291 IE O7
\(.12185748 \mathrm{E} \quad 08\)
\(.15277855 E 08\)
\(.21663648 \mathrm{E} \quad 08\)
.22228613508
\(.18539144 E 10\)
\begin{tabular}{cccccccc}
1 & 207 & \(.89440000 E 04\) & \(.63586000 E 05\) & \(.12390663 E 09\) \\
4 & 1215 & \(.59123000 E 05\) & \(.06297000 E 05\) & \(.88747330 E 09\) \\
5 & 54 & \(.24730000 E 04\) & \(-.34561000 E 05\) & \(.42873229 E 08\) \\
0 & 492 & \(.19385000 E 05\) & \(-.61146500 E 06\) & \(.33522815 E 09\)
\end{tabular}

\begin{tabular}{rr}
.70698243 E OL & .11716060 E 03 \\
.11213402 E O1 & \(.12251273 \mathrm{E} \mathrm{O3}\) \\
\(-.13975333 \mathrm{E} \mathrm{O2}\) & .13092447 E 03 \\
\(-.31543203 \mathrm{E} \mathrm{O2}\) & .12766440 E 03 \\
HSS/TSS \(=\) & .01274
\end{tabular}
.13259253508 \(.13839046 E 08\) .23615609 C 8 \(.2324188 C E\) GB \(.18534139 E 10\)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline MEAN & & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{\[
\begin{gathered}
\text { STO. OEV. } \\
.10194529 E 03
\end{gathered}
\]}} & \multicolumn{3}{|l|}{\multirow[t]{2}{*}{\(B \quad 5 \quad 5\)}} \\
\hline .48265373E & 02 & & & & & \\
\hline & & & & \multicolumn{3}{|l|}{. 11805314 E 08} \\
\hline . 19885095 E & 01 & . 14.985230 E & 03 & & & \\
\hline & & & & \multicolumn{3}{|l|}{.41746346E 07} \\
\hline \multirow[t]{2}{*}{. 12500503 E} & 01 & .12382681E & 03 & & & \\
\hline & & & & \multicolumn{3}{|l|}{. 574407C3E 07} \\
\hline \multirow[t]{2}{*}{-. 65037756 E} & 01 & . 14676769 E & 03 & \multicolumn{3}{|l|}{\multirow[t]{2}{*}{\[
.54425190 E \quad 07
\]}} \\
\hline & & & & & & \\
\hline \multirow[t]{2}{*}{-. 10926499 E} & 02 & .15033769 E & 03 & \multicolumn{3}{|l|}{\multirow[b]{2}{*}{. 51517805E 07}} \\
\hline & & & & & & \\
\hline \multirow[t]{2}{*}{-. 14561882 E} & 02 & .11469117E 0 & 03 & & & \\
\hline & & & & \multicolumn{3}{|l|}{\(.18539136 i 10\)} \\
\hline
\end{tabular}

TRY ON VARIABLE 16 QVER GRDUP 1 . RESULTS FOLLOW.

\begin{tabular}{|c|c|c|c|}
\hline Mean & & \multicolumn{2}{|r|}{DEV.} \\
\hline . 12151425 E & 02 & . 12335702 E & 03 \\
\hline . 10235428 E & 02 & . 14800839 E & 03 \\
\hline . 97679333 E & 01 & . 13555915 & 03 \\
\hline -. 27670216 E & 01 & . \(11856023 E\) & 03 \\
\hline -. 32633650 E & 01 & .83350591E & 02 \\
\hline -. 22858007 E & 02 & .66382153E & 02 \\
\hline -. \(30311526 E\) & 02 & . 10963858 E & 03 \\
\hline -. 45898072 E & 02 & . 10808981 E & 03 \\
\hline BSS/T & S \(5=\) & . 01611 & \\
\hline
\end{tabular}

P \(S\)
. 39901565 E 07
. 11159376 É 08
\(.19377754 E O 8\)
\(.22228613 E 08\)
.29865566 E 08
.27674167508
.27011464 E 08
\(.18539143 E 10\)

TRY ON VARIABLE 17 DVEK GROUP 1 . RESULIS FOLLOWE
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline code & N & SUM OF WEIGHT & SUM OF & \(Y\) & \multicolumn{2}{|l|}{SUM Y-SGUARE} \\
\hline 2 & 518 & . \(24330000 \dot{E} 05\) & . 46236200 E & 06 & . 36813924 E & 09 \\
\hline 3 & 327 & . 15699000 ES & . \(27440900 E\) & 06 & . \(27255267 E\) & 09 \\
\hline 1 & 461 & . \(21089000{ }^{\text {c }} 05\) & -. 33520000 E & 04 & . 31129267 E & 09 \\
\hline 0 & 1240 & . 55650999 EL & -. 72427399 E & 06 & . 901929755 & 09 \\
\hline
\end{tabular}
MEAN

STD. DEV.
\(.12153164 E 03\)
\(.13059716 E 03\)
\(.12149435 E 03\)
\(.12663930 E 03\)
\(B \quad S \quad S\)
\(.11007977 E 08\)
\(.20459374 E 08\)
\(.18226464 E 08\)
.18539136 E 10
- FOR VARIABLE 17 ( INCOME COMM , B \(S S=.20459374 E 08\) TRY ON VAKIABLE 18 OVER GRTUP \(I\) - RESULTS FOLLOW.

- FQR VARIABLE 18 (ALSIL TO COMM) P S S = . \(14359911 E\) OB
MEAN
\(.53235975 E\) O1
\(-.22058357 E 02\)
\(-.40468213 E 02\)
\(.16489926 E 02\)
STO. DEV.
.126492 ITE O3
\(.12079805 E 03\)
\(.97714500 E 02\)
\(.17866555 E 03\)

A \(S S\)
\(.14359911 E 08\)
.37276733507
.25605632 E 06
\(.18539110 E 10\)
try on variable 19 over group 1 . results follow.
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline CODE & N & \multicolumn{2}{|l|}{SUM DF WEIGHT} & SUM OF & Y & \multicolumn{2}{|l|}{y-SQUARE} \\
\hline 1 & 348 & . 17082000E & 05 & . 13970800 E & 06 & . 24201426 E & 09 \\
\hline 2 & 462 & . \(22693000 \ddot{z}\) & 05 & . 12220700 E & 06 & . 28641351 E & 09 \\
\hline 3 & 310 & .14160000 c & 05 & . 17119100 F & 06 & . 28577450 E & 09 \\
\hline 4 & 471 & . 22191000 L & 05 & . 22851000 E & 05 & . 32492352 E & 09 \\
\hline 5 & 375 & . 17819 CQOE & 05 & .54461300 E & 06 & . 28852334 E & 09 \\
\hline 6 & 580 & . 22824000 E & 05 & -. 791424995 & 06 & . 42626606 E & \\
\hline
\end{tabular}
* FOR VARIABLE 19 ( SIZE OF PLAC) B S \(S=.53721296 E 08\)


B S S
.13129112107
\(.2553 \mathrm{B114E} \mathrm{\quad 07}\)
\(.63377886 E 07\)
\(.76423027 E 07\)
\(.53721296 E 08\)
.18539144 E10

* FOR VARIARLE 20 (H-F ED DIFF) BSS \(=.43842943 E 07\)
\begin{tabular}{cc} 
MEAN & STO. DEV. \\
\(.93221218 E\) O1 & \(.13369325 E 03\) \\
\(.62837071 E ~ 01\) & \(.11489826 E 03\) \\
\(-.41596117 E ~ 01\) & \(-14126503 E 03\) \\
\(-.42861694 E 01\) & \(.11948460 E 03\)
\end{tabular}
\(B \quad 5 \quad 5\)
\(.36446630 E 07\)
\(.43842943 E 07\)
\(.27006947 E 07\) .18539133510


\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CANDIDATE & GROUP S & ARE AS FOLLOWS. & & & & \\
\hline Group & N & TOTAL WEIGHI & SUM OF Y & SUA Y-SQUARE & \(T\) S & 5 \\
\hline 2 & 1821 & .80918000E 05 & . 18264970E 07 & . 14236475 E 10 & . 13861287 E & 10 \\
\hline 3 & 725 & . 21851000 E & -. \(18173520 E 07\) & .43026788 O 09 & . 31168081 E & 09 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline S TEP & NO. & 2 & & PA & & & & 2 & & & \\
\hline FOR & VARIABLE & 2 & 1 & GEOG MOBILIt & , & B & S & S & & . 90724754 E & 07 \\
\hline FOR & VARIABLE & 3 & 1 & EUUCATİN & J & A & 5 & S & & . \(18968550 E\) & 07 \\
\hline FOR & VARIABLE & 4 & 1 & IMMIGRATIDN & ) & B & 5 & S & & . 51775050 E & 06 \\
\hline FOR & VARIABLE & 5 & 1 & olicupation & 1 & B & S & S & & . 13558486 E & 08 \\
\hline FOR & VARIABLE & 6 & 1 & SUPR RESP & 1 & \(B\) & S & S & & . \(10490296 E\) & 08 \\
\hline FOR & VARIABLE & 7 & ( & FKEO UF UNEM & ) & \(B\) & S & S & & . 93321775 E & 07 \\
\hline FOR & VARIABLE & 9 & 1 & REL X ATTEND & ) & E & 5 & S & & . \(85667425 E\) & 07 \\
\hline FOR & VARIABLE & 10 & 1 & WURK X N/ACH & 1 & B & S & S & & . \(28859635 E\) & 07 \\
\hline FOR & VARIABLE & 11 & 1 & RACE & 1 & H & S & S & & . \(38651645 E\) & 07 \\
\hline FDR & VARIABLE & 13 & 1 & H-W ED DIFF & ) & \(B\) & S & S & & . 263116225 & 08 \\
\hline FOR & VARIABLE & 14 & & UKB-RUR MTG & 1 & \(B\) & S & S & & . 10759083 E & 08 \\
\hline FOR & VARIABLE & 15 & 1 & N-S MIG & ) & 8 & S & S & & . 70148360 E & 07 \\
\hline FOR & VARIABLE & 16 & 1 & FAM COMP & 1 & \(B\) & S & S & & . \(23450021 E\) & 08 \\
\hline FOR & VARIABLE & 17 & 1 & INCOME COMM & ) & B & S & S & \(=\) & . \(77834205 E\) & 07 \\
\hline FOR & VARIABLE & 18 & 1 & ABIL TO COMM & 1 & B & 5 & S & \(=\) & . 286192358 & 07 \\
\hline FOR & VARIABLE & 19 & 1 & SIZE OF PLAC & ) & B & S & 5 & & . 95129550 E & 07 \\
\hline FOR & VARIABLE & 20 & 1 & H-F ED DIFF & ) & B & S & S & = & .12748730 E & 07 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline BSS/TSS = & . 00655 \\
\hline GSS/TSS & . 00137 \\
\hline BSS/TSS & .00037 \\
\hline BSS/TSS & . 00978 \\
\hline BSS/TSS & . 00757 \\
\hline BSS/ISS & . 00673 \\
\hline BSS/TSS & . 00618 \\
\hline BSS/TSS & . 00208 \\
\hline BSS/TSS & .00279 \\
\hline RSS/TSS & . 01898 \\
\hline BSS/TSS & . 00776 \\
\hline BSS/TSS & . 00506 \\
\hline BSS/TSS & . 01692 \\
\hline HSS/TSS & .00562 \\
\hline BSS/TSS & . 00206 \\
\hline BSS/TSS & . 00686 \\
\hline BSS/TSS & . \(\mathrm{COO992}\) \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline 3 & 496 & . 23911000 E & 05 & . 64921000 E & 06 & . 39837207 E & 09 & . 27151102 E & 02 & . 12618810 E & 03 & .19245623 E 08 \\
\hline 4 & 264 & .13057000 c & 05 & \(.35158500 E\) & 06 & . 24668512 F & 09 & . 26926935 E & 02 & . 13478828 E & 03 & . 26311622 E.08 \\
\hline 5 & 251 & . \(12260000 t\) & 05 & . 49329000 E & 05 & . 24150190 E & 09 & . \(40235725 E\) & 01 & . 14029316 E & 03 & \\
\hline 0 & 408 & . \(20368000{ }^{\circ}\) & 05 & -. 11631300 E & 06 & . 17548545 E & 09 & -. 57105754 E & 01 & . \(92645194 E\) & 02 & .13861287E 10 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline CANDIDATE & GROUPS & ARE AS FOLLOWS. & & & & \\
\hline GROUP & N & TOIAL WEIGHT & SUM OF Y. & SUM. Y-SQUARE & 15 & 5 \\
\hline 3 & 725 & . \(27851000 E 05\) & -. 18173520E 07 & .43026788E 09 & . 31168081 E & 09 \\
\hline 4 & 1162 & . 56290000E 05 & .18934810 O & . 10066601E 10 & . 94296728 E & 09 \\
\hline 5 & 659 & . \(3 \angle 628000 \mathrm{E} 05\) & -.66983999E 05 & .41698735E 09 & . \(41684983 E\) & 09 \\
\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline 3 & 139 & . \(63670000 E\) & 04 & . 25668900 E & 06 & . 30022628 E & 09 & . 40315533 E & 02 & . 21337326 E & 03 & & \\
\hline & & & & & & & & & & & & . 98246095 E & 07 \\
\hline 4 & 146 & . 73249999 E & 04 & . 145753006 & 06 & .11996997E & 09 & . 19898020 E & 02 & . 12642081 E & 03 & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{\[
.60405545 E 07
\]}} \\
\hline . 6. & .. 355 & . 17.007000E & 05 & . 30432500 E & 06. & 13630499 E & 09 & E & & . \(87717967 E\) & 02 & & \\
\hline & & & & & & & & & & -年717967E & & \multicolumn{2}{|l|}{.94296738 E 09} \\
\hline
\end{tabular}

CÁNDIDATE GROUPS ARE AS FOLLOWS.



\footnotetext{
FAILED TO SPLIT GROUP O TRIED ON VARIABLE 19 , BUT ASS \(=.89391670 E 07{ }^{\circ}\)
}


FAILED TO SPLIT GROUP \(\quad 5\) TRIEO ON VARIABLE 7 , BUT BSS \(=.89513274 E 07\)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline CANDIDA & E GROUPS & ARE & \(E\) AS FOLLOWS. & & & & & & & & & & & \\
\hline GROUP & N & & TOTAL WEIGHT & & & & & DF Y & & SUM Y-SQU & Iare & & 1 S & S \\
\hline 3 & 725 & & . 27851000 E 05 & & & -. 1 & 8173 & S20E 07 & & . 43026788 E & 09 & & . \(31168081 E\) & 09 \\
\hline 7 & 501 & & .24332000 E 05 & & & & 5007 & B00E O6 & & . \(25627496 E\) & 09 & & . \(24794970 E\) & 09 \\
\hline - FOR & VARIABLE & 2 & GEOG MOBILIT & ) & A & S & S = & .14394610 E & 07 & & BSS/TSS & \(=\) & . 00462 & \\
\hline - FOR & VARIABLE & 31 & ( EUUCATIDN & , & B & S & S \(=\) & . 21420190 E & 07 & & BSS/TSS & & . 00687 & \\
\hline - FOR & VARIABLE & 41 & ( IMMIGRATIIN & ) & B & S & \(S=\) & . 11012900 E & 06 & & BSS/TSS & & . 00035 & \\
\hline - FOR & VARIABLE & 51 & 1 OlCupation & ) & B & S & \(S=\) & .19986978E & 08 & & BSS/TSS & & . 06413 & \\
\hline - FOR & VARIABLE & 61 & ( SUPR KESP & ) & 8 & S & & . 12670401 E & 08 & & GSS/TSS & \(=\) & . 04065 & \\
\hline - FOR & VARIABLE & 71 & FKEQ OF UNEM & 1 & B & S & \(S=\) & . 72915199 E & 07 & & BSS/TSS & \(=\) & . 02339 & \\
\hline - FOR & VARIABLE & 91 & \((\) REL X ATTEND & 1 & B & S & S & . 20821010E & 07 & & BSS/TSS & = & . 00ヶ68 & \\
\hline - FOR & VARIABLE & 10' & ( WURK \(\times\) N/ACH & ) & 8 & 5 & S = & . 26076620 E & 07 & & PSS/TSS & \(=\) & . 00837 & \\
\hline - FOR & VARIABLE & 111 & ( RACE & ) & 8 & S & S & . 29192000 E & 05 & & ESS/TSS & \(=\) & -.00009 & \\
\hline - FOR & VARIABLE & 131 & ( H-W ED DIFF & , & B & 5 & & . 16826890 E & 07 & & BSS/TSS & & . 005140 & \\
\hline - Fur & VARIABLE & 141 & ( UKB-RUR MTG & , & 8 & S & S = & . 14023486 E & Ot & & BSS/TSS & \(=\) & . 04479 & \\
\hline - for & VARIABLE & 151 & \(1 \mathrm{~N}-\mathrm{S}\) MIG & ) & A & S & \(S=\) & . 77869079 E & 07 & & BSS/TSS & \(=\) & . 02498 & \\
\hline - FOR & VARIABLE & 151 & 1 FaM COMP & ) & B & S & S & . 25444590 E & 07 & & BSS/TSS & \(=\) & . 00816 & \\
\hline - FOR & VARIABLE & 171 & ( IINCOME COMM & 1 & B & S & \(\mathrm{S}=\) & . 19726020 E & 07 & & BSS/TSS & \(=\) & . 00633 & \\
\hline - FOR & VARIABLE & 181 & \((\) ABIL TO COMM & 1 & \(B\) & S & S & . 20171800 E & 06 & & BSS/TSS & \(=\) & . 00065 & \\
\hline - FOR & VARIABLE & 191 & ( SizE OF Plac & , & \(B\) & 5 & \(\mathrm{S}=\) & . 13019936 E & 08 & & BSS/ISS & & . 04.177 & \\
\hline - FOR & VARIABLE & 201 & ( H-F ED DIFF & 1 & A & S & S = & . 53018200 E & 06 & & BSS/TSS & \(=\) & .00170 & \\
\hline
\end{tabular}

DECOMPOSE GROUP 3 INTU GROUP 8 ANO 9 BY VARIABLE 5 IN S TEP 4.
CODE \(N\) SUM OF WEIGHT SUM OF Y SUM Y-SGUARE MEAN STD. DEV.


\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline GROUP & N & & TOTAL WEIGHT & & & & & OF \(Y\) & & SUM Y-SQU & ARE & & 1 S & S \\
\hline 8 & 528 & & . 2U728000E 05 & & & -. 1 & 10270 & 480E 07 & & . 23167940 E & 09 & & .180790385 & 09 \\
\hline 9 & 197 & & . 71230000E 04 & & & & 79030 & 400E 36 & & . 19858848 E & 09 & & . 11090346 E & 09 \\
\hline FOR & VARIABLE & 2 & ( GeOg mosilit & , & B & S & S = & . 90803350 E & 06 & & 8SS/TSS & & . 00502 & \\
\hline FOR & VARIABLE & 3 & EUUCATION & 1 & \(B\) & S & \(S \geq\) & . 31286315 E & 07 & & BSS/TSS & \(=\) & . 01731 & \\
\hline FOR & VARIABLE & 4 & ( IMMIGRATION & , & 8 & S & \(5 \geq\) & . 82016999 E & 05 & & BSS/TSS & = & . 00045 & \\
\hline FOR & VARIABLE & 5 & 1 OCCUPATIDN & 1 & B & S & \(S=\) & . 16452200 E & 06 & & BSS/TSS & \(=\) & .00091 & \\
\hline FOR & VARIABLE & \(\bigcirc\) & I SUPR RESP & ) & \(B\) & S & \(S=\) & . 526544505 & 06 & & ESS/TSS & & .00271 & \\
\hline FDR & VARIABLE & 7 & I FREQ OF UNEM & ) & B & S & \(5=\) & . 10977140 E & 07 & & RSS/TSS & \(=\) & . 00607 & \\
\hline FOR & VARIABLE & 9 & 1 REL X ATTEND & ) & B & 5 & \(5=\) & . 208883 ¢OE & 07 & & BSS/TSS & & . 01155 & \\
\hline FOR & VARIABLE & 10 & ( WURK X N/ACH & 1 & H & S & \(S \geq\) & . 21217685 E & 07 & & BSS/TSS & \(=\) & .01174 & \\
\hline FOR & VARIABLE & 11 & ( RACE & 1 & B & S & \(S=\) & . 54616400 E & 06 & & BSS/TSS & \(=\) & .00302 & \\
\hline FOR & VARIABLE & 13 & ( H-W ED DIFF & 1 & B & S & S \(=\) & . 40359760 E & 07 & & BSS/TSS & \(=\) & . 02232 & \\
\hline FOR & VARIABLE & 14 & ( UK8-RUR MTG & ) & B & S & \(S=\) & . 66580199 E & 07 & & BSS/TSS & & . 03683 & \\
\hline FOR & VARIABLE & 15 & \(1 \mathrm{~N}-\mathrm{S}\) MIG & \()\) & B & S & \(S=\) & . \(44462625 E\) & 07 & & BSS/TSS & \(=\) & . 02487 & \\
\hline FOR & VARIABLE & 16 & 1 FAM COMP & 1 & 8 & 5 & \(S=\) & . 26090160 E & 07 & & BSS/TSS & \(=\) & . 01443 & \\
\hline FOR & VARIABLE & 17 & 1 INCOME COMM & ) & 8 & S & S = & . 41310300 E & 06 & & BSS/TSS & \(=\) & . 00228 & \\
\hline FOR & VARIABLE & 18 & I ABIL TO COMM & ) & 8 & S & \(S=\) & . 46914350 E & 06 & & BSS/ISS & & . 00259 & \\
\hline FOR & VARIABLE & 19 & ( SILE OF PLAC & 1 & 8 & S & \(S=\) & . 23942800 E & 07 & & BSS/TSS & & . 01.324 & \\
\hline FQR & VARIABLE & 20 & 1 H-F ED DIFF & ) & \(\dot{\text { B }}\) & S & \(S=\) & . 11693665 E & 07 & & HSS/TSS & & . 00647 & \\
\hline
\end{tabular}

FAILED TO SPLIT GROUP 8 TRIED ON VARIARLE 14 ; BUT RSS \(=.66580199 E 07\)


FAILEO TO SPLIT GRQUP 9 TRIEO ON VARIABLE 15 , BUT BSS = . \(56207870 E 07\)
GROUP N TOTAL WEIGHT SUM OF Y SUM Y-SQUARE . S

THAT IS ALL. NO MORE GROUPS ARE AVAILABLE. FINAL S TE P NO. IS 5 NO. OF GROUPS \(A R E\)
* THIS IS THE END OF 2ND CORE.

TIME IS NOW 12. 15. 21.18.
dependent variable 28 ( residuals 5il
WEIGHTED BY VARIABLE 26
* total Group

TOTAL WT \(\begin{aligned} & \text { N } \\ & \text { SUM }=\end{aligned}\)
STO. MEAN \(=0.78317018 E-01\)
SUM \(Y=.91450000 \mathrm{E} 04\)
SUM Y SO. \(=.18539100 \mathrm{E} 10\)
- GRDUP NO.
2 SPLIT FROM GROUP 1 ON VARIABLE 5 IOCCUPATION
PREDICTOR
INCIUDED ARE

VALUES OF
1821
WEIGHT SUM \(=\)
PCT OF TOTAL =
76.1
\[
\text { STO NEAN }=.20541364 E N_{0}^{0}
\]

STO. DEVV \(=.12485529 \mathrm{~F}_{0}\)
GROUR UEVIATION \(=.20463046 E .02\) TSS(I) \(=.13861287 E 10\) WTD. MEAN SQ. \(=.37518739 E 08 \quad\) (TSS(S)/TSS(T)) \(=.74767885 E 00\)
- GROUP No. VALUES OF

3 SPLIT
PREDICTOR
\[
\begin{aligned}
\text { FRIM GROUP } 1 & \text { ON VARIABLE } \\
\text { INCLUUED ARE } & 0 \\
\text { MEAN } & =-.65252665 E 02 \\
\text { STD. DEV. } & =-10578757 E 03 \\
\text { WTD. MEAN SO. } & =.11858706 E 09
\end{aligned}
\]
inccupation
725
27951
GROUP DEV
EVIA

27951
WEIGHT SUM =
\[
\text { IATION }=-.65330982 E 02
\]
\[
\text { TSSII) }=.31168081 E 04
\]

PCT OF TOTAL =
23.9
\[
\text { (ISS(I)/TSS(T)) }=.16812086 \mathrm{E} .00
\]
- GROUP No. VALUES OF 4 SPLIT PREDICTOR

1162 WEIGHT SUM = PCT DF TOTAL =

FRDM GROUP 2 ON VARIARLE 13 (H-W ED DIFF)
INCLUOED ARE 1 \begin{tabular}{llcll}
2 & ON ARIARLE & 13 & 1 H- \\
\hline
\end{tabular}
STD. DEV. \(=.12942 .932 \mathrm{E} 03\)
GROUP DEVIATION \(=.33559646 E 02\)
TSS(1) \(=.94296728 \mathrm{E}\) 09
WTD. MEAN SQ. \(=.63692845 \mathrm{OB} \quad\) OTSSIII/TSSIT) \(=.50863724 \mathrm{E} 00\)
- GROUP ND. 5 SPLIT FRMM GRIJUP 2 ON VARIABLE 13 (H-H ED DIFF)

VALUES OF PREDICTOR INCLUDED ARE 0 5
** THIS GRDUP IS RETAINED AS ONE OF FINALS.
\begin{tabular}{rlrl}
N & \(=\) & 659 & 32628
\end{tabular}\(\quad\)\begin{tabular}{rl} 
HEIGHT SUM & \(=\)
\end{tabular}\(\quad\) STO. DEV \(=-.20529606 E 01\)

GROUP DEVIATION \(=-.21312776 E 01\)
TSS(I) \(=.41684983 E 09\)

\(\begin{aligned} \text { TSS(I) } & =.41684983 E 09 \\ (T S S(1) / T S S(T)) & =.22484910 E 00\end{aligned}\)
 VALUES OF PREDICTOR INCLUDED ARE 1 2 3

WEIGHT SUM \(=\quad 31.958 \quad\) STO. DEV. \(=\quad .14642549[03\)
GROUP OFVIATION \(=.45087306 E 02\)
TSS(I) \(=.68519307 \mathrm{E} 09\)
(TSS(I)/TSS(T)) \(=.36959364 E 00\)
- GROUP NO. 7 SPLIT FROM GROUP 4 ON VARIABLE 5 (OCCUPATIUN,

VALUES OF. PREDICTOR INCLUDED ARE 4 GIN 6
\(\begin{array}{rrrrrr}\text { ** THIS IGROUP IS RETAINED AS ONE OF FINALS: } \\ N & \text { SOl } & \text { MEAN }= & .18497369 E 02 \\ \text { HEIGHT SUM }= & 24332 & \text { STO. DEV. }= & .10094688 \mathrm{E} 03\end{array}\)
WEIGHT SUM \(=\quad 24332 \quad\) STO. DEV. \(=.10094688 E 03\)

PCT OF TOTAL \(=20.8\) WTD. MEAN SQ. \(=.83252590 E 07\)
- GROUP NO. B SPLIT FROM GRIJUP 3 ON VARIABLE 5 IOCCUPATION
... THIS GROUP IS RETAINED AS ONE OF FINALS.
\(N=528\) MEAN \(=-.49548922 E 02\)
HEIGHT SUM \(=\quad 20728 \quad\) STO. DEV \(=.93391845 E .02\)
PCT OF TOTAL \(=17.8\) WTD. MEAN SO. \(=.50 B R 9018 E O 8\)

GROUP DEVIATION \(=.18419052 E \ldots\)
TSS(1) \(=.24794970\) O4 \((T S S(1) / T S S(T))=.13374425 E 00\)
* GRDUP NO. 9 SPLIT FROM GRIUP 3 ON VARIABLE 5 (OCCUPATIDN.

M Y Y \(=.18934810 \mathrm{E} 07\) SUM Y SO. \(=.10066601 E 10\)

SUM \(Y=.18264970 E 07\) SUM Y SU. \(=.14236475 \mathrm{E} 10\)

\author{
SUM \(Y=-\therefore 18173520 E 07\)
} SUM Y SQ. \(=.43026788 \mathrm{E} 09\) SUM Y SO. \(=.41698735 E\) O9

SUM Y SO \(=.14434030 E\) O7

SUM Y \(=.45007800 \mathrm{E} 06\) SUM Y SO. \(=.25627496 \mathrm{E} 09\)

SUM \(Y=-.10270480 E 07\) SUM Y SQ. \(=.23167940 \mathrm{E} 09\)

VALUES OF PREDICTOR INCLUDED ARE 8


GROUP DEVIATION \(=-.11102932 E 03\) TSSII \(=.11090346 \mathrm{E}\) 69 (TSS(I)/TSS(T)) = .59821408E-01

SUM \(Y=-.79030400 E\) OG SUM Y SU. \(=.19858848 \mathrm{E}\) O9
\begin{tabular}{|c|c|c|c|c|}
\hline & ANALYS & variance & * * & \\
\hline SOURCE OF & SUm of & degree of & mean & \\
\hline VARIATION & squares & freeoom & square & F \\
\hline TOTAL & .1853.9092E 10 & 116723 & & \\
\hline between & . 21222829E 09 & 4 & . 53057072 E 08 & . 37722163 E 04 \\
\hline WITHIN & . 16416809 E 10 & 116719 & . 14065225E 05 & \\
\hline
\end{tabular}

RESIDUALS ARE NOT REQUESTED.

TIME IS NOW 12. 15. 23. 15.
**** all inpur data have been processed.
AT LOC 75077

PUBLICATIONS AND RESEARCH REPORTS MAKING USE OF THE ALD PROGRAM OR REFERRING TO THE DETECTION OF INTERACTION EFFECTS, SRC MONOGRAPH NO. 35 OR JASA REPRINT 58, (JUNE, 1963) PROBLEMS IN THE ANALYSIS OF SURVEY DATA, AND A PROPOSAL

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[^0]:    SOURCE
    ISR Project 719
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[^1]:    *See (27) for a description of these coefficients. **Beta coefficients do not add. This is an adjusted $R^{2}$.
    Source: ISR Study 678, Deck 33

[^2]:    *If $W$ is small, say $W<50$, and the run is unweighted, then it may be advisable to correct for small sample sizes and $\sigma_{a d j}=\sqrt{N /(N-1)}$ where N is the number of observations over which summation has taken place.

[^3]:    *The major hint needed in going from (a) and (b) to (5) and (6) is to replace

    $$
    \left[\Sigma(m)+\Sigma\left(L_{m}\right)\right]^{2} \text { by }\left[\Sigma(m)+\Sigma\left(I_{m}\right)\right]\left[\Sigma\left(P_{n}\right)+\Sigma\left(P_{n}\right)-\Sigma\left(H_{m}\right)\right]
    $$

    and to replace

    $$
    \left[\Sigma\left(P_{n}\right)\right]^{2} \text { by }\left[\Sigma\left(P_{n}\right)\right]\left[\Sigma(m)+\Sigma\left(I_{m}\right)+\Sigma\left(H_{m}\right)-\Sigma\left(P_{m}\right)\right]
    $$

    etc.

[^4]:    *These decision rules constitute the crucial steps in the algorithm.

[^5]:    - The authors are indebted to many individualn for advice and improvementa. In particular. Professor L. J. Savago noticed that some interactions would remain hidden, and Professor William Ericson proved that locating the best combination of subclaspes of a aingle code was aimple enough to incorporate into the program. A Ford Foundstion grant to the Depertment of Economice of the University of Michigan eupported the author's work on eome aubatantive problems which led to the present focus on methods. Support from the Rockefallar Foundation th aleo gratofully scknowledged.
    **Reprinted by permission from the Journal of the American Statistical Association, 58 (June 1963), 415-35.

[^6]:    ' Onc secmingly appropriate measure for two clasaifications both being used to predict the same variable is one called lambda suggested by Goodman and Kruakal. With many kinds of survey data this measure, which aseumea that an absolute prediction has to be made for each individual, is too ingensitivo to deal with situations where each class on the predicting charactenatic has the same modal class on the other characteristic that is to be predicted. An effective and properly atochastic measure would be derived by assigning a one-xero dummy variable to beloaging to each class of each of the two characteristics and then computing the canonical correlation between the two sets of dummy variables.

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    I In other words family composition had different effects on different expenditures. F. G. Fontythe, "The relationship letween family size and farmily expeaditure," Journal of the Royal Statiatical Society, Seriea A, vol. 123 (1081). 367-07, quote from p. 388.

[^7]:    Source: 1059 Survoy of Consumer Finances.

[^8]:    f For an excellent atstement of the spplication of this problem to the aconomists' concern with the permanent income hypotheais veraus the relative income hypothesis, see Jean Crockatt. "Liquid asseta and the theory of consumption" (New York: National Buresu of Economic Research, 1982) (mitneographed).

[^9]:    - For a diacussion of alternative strategies made while commenting on a mories of papera, see James Morgan. "Commenta," in Conmmption and Saving, Volume I, I. Friend and R. Jones (Editors.) (Phitadelphia: University of Penneylvania Press, 1960), pp. 278-84.
    * Charles Wettof and others, Family Planning in Medropolitan America (Princeton: Princeton University Prese, 1861).

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